CBSE PAPER - 2022-23 (STANDARD) MATHEMATICS

ANSWER AND SOLUTIONS

SECTION-A

1. Option (2)
1

2. Option (2)
does not exist

3. Option (3)
0

4. Option (4)
2 : 1

5. Option (1)
47.5

6. Option (3)
3

7. Option (1)
1

8. Option (2)
2

9. Option (1)
–1

10. Option (2)
3 cm

11. Option (3)
30 – 40

12. Option (3)
550 cm^2

13. Option (1)
$2\sqrt{10}$ units

14. Option (4)
$5(9x^2 - 4)$

15. Option (2)
Increases by 3

16. Option (1)
p + q = 1

17. Option (3)
480

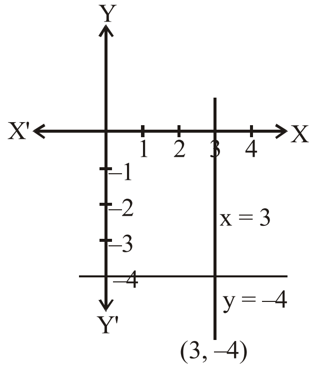
18. Option (1)
$\frac{3}{4}$

19. Option (2)
Both Assertion (A) and Reason (R) are true and Reason (R) is not the correct explanation of Assertion (A).

20. Option (4)
Assertion (A) is false but Reason (R) is true.

SECTION-B

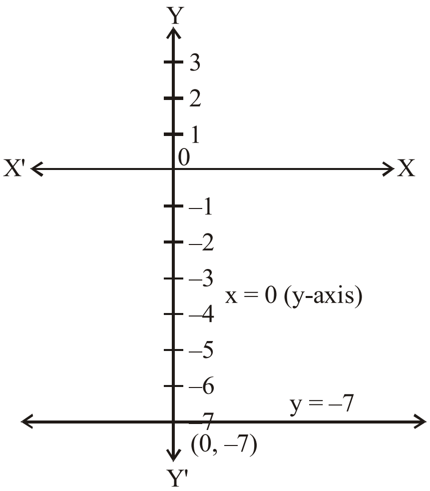
21. (a) x = 3 and y = –4 meet at (5, 7)



Thus, x = 3, y = 4

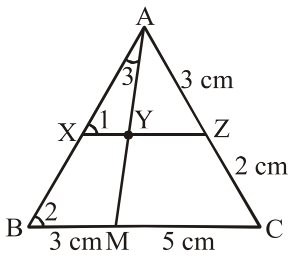
OR

(b) Pair of equation x = 0 (y-axis) and y = –7 is consistent.



Thus, x = 0, y = –7
So, pair of linear equation is consistent.

22.



In ΔABC

$XZ \parallel BC$

So, by Thales theorem

$$\frac{AZ}{ZC} = \frac{AX}{BX} = \frac{3}{2}$$

Now in ΔABM and ΔAXY

$XY \parallel BM$ $\quad (\because XZ \parallel BC)$

$\angle 1 = \angle 2$ $\quad$ (corresponding $\angle$s)

$\angle 3 = \angle 3$ $\quad$ (common)

$\Delta AXY \sim \Delta ABM$

$$\frac{AX}{AB} = \frac{XY}{BM} \quad .....(1)$$

Now $\dfrac{AX}{BX} = \dfrac{3}{2} \Rightarrow \dfrac{BX}{AX} = \dfrac{2}{3}$

$$1 + \frac{BX}{AX} = \frac{2}{3} + 1$$

$$\frac{AX + BX}{AX} = \frac{2+3}{3}$$

$$\frac{AB}{AX} = \frac{5}{3}$$

$$\frac{AX}{AB} = \frac{3}{5} \quad .....(2)$$

From (1) and (2)

$$\frac{AX}{AB} = \frac{XY}{BM}$$

$$\frac{3}{5} = \frac{XY}{3}$$

$$XY = \frac{3 \times 3}{5} = \frac{9}{5}$$

$XY = 1.8$ cm

23. (a) $\sin\theta + \cos\theta = \sqrt{3}$

squaring both sides

$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$

$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$1 + 2\sin\theta\cos\theta = 3 \quad \{\sin^2\theta + \cos^2\theta = 1\}$

$2\sin\theta\cos\theta = 3 - 1$

$2\sin\theta\cos\theta = 2$

$\sin\theta.\cos\theta = 1$

OR

(b) If $\sin\alpha = \dfrac{1}{\sqrt{2}}$ and $\cot\beta = \sqrt{3}$

$$\text{cosec}\,\alpha = \frac{1}{\sin\alpha} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}$$

$\text{cosec}^2\beta = 1 + \cot^2\beta$

$\text{cosec}^2\beta = 1 + (\sqrt{3})^2$

$\text{cosec}^2\beta = 1 + 3$

$\text{cosec}\,\beta = \sqrt{4}$

$\text{cosec}\,\beta = 2$

So, $\text{cosec}\,\alpha + \text{cosec}\,\beta = \sqrt{2} + 2$

24. Greatest number which divides 85 and 72 leaving remainders 1 and 2 respectively will be HCF of 84(85 – 1 = 84) and 70(72 – 2 = 70)

HCF of 70 and 84 = 14

25. Number of Red Ball = 4 Balls

Number of Blue Ball = 3 Balls

Number of Yellow Ball = 2 Balls

Total number of Balls = 9 Ball

(i) Probability of drawing Red ball

$$= \frac{\text{Number of Red balls}}{\text{Total number of balls}} = \frac{4}{9}$$

(ii) Probability of drawing Yellow ball

$$= \frac{\text{Number of Yellow balls}}{\text{Total number of balls}} = \frac{2}{9}$$

SECTION-C

26. Let the two numbers be x and y such that x > y

According to problem

$$\frac{1}{2}(x - y) = 2$$

$\Rightarrow x - y = 4$ ....(1)

Also, $x + 2y = 13$ ....(2)

Subtract (1) from (2)

$$\begin{array}{l} x + 2y = 13 \\ \underline{\overset{-}{x} \overset{+}{-} y = \overset{-}{4}} \\ 3y = 9 \\ y = 9/3 = 3 \quad \Rightarrow y = 3 \end{array}$$

Put y = 3 in (1)

$x - 3 = 4$

$x = 7$

Thus, number are 7 and 3.

27. Let $\sqrt{5}$ is a rational number

$\sqrt{5} = \frac{p}{q}$ [p and q are co-prime]

Squaring both sides

$$5 = \frac{p^2}{q^2}$$

$5q^2 = p^2$ ....(1)

5 divides p^2

5 divides p also

So, p = 5r

Put p = 5r in (1)

$5q^2 = (5r)^2$

$5q^2 = 25r^2$

$q^2 = 5r^2$

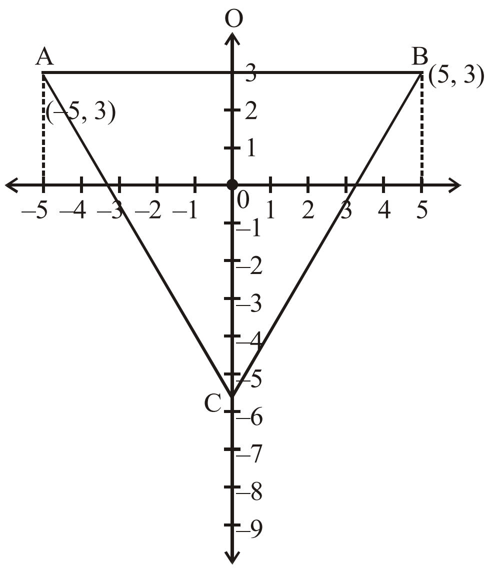
5 divides q^2

5 divides q.

But p and q are mutually co-prime which is a constradiction, hence our assumption is wrong

$\sqrt{5}$ is irrational number.

28.



AB = 10 cm

ΔABC is equilateral triangle

So, AC = BC = 10 cm

In ΔACO

By pythagoras theorem

$AC^2 = AO^2 + OC^2$

$(10)^2 = (5)^2 + OC^2$

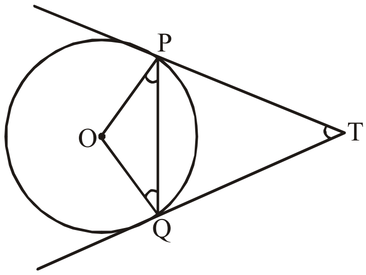
$OC^2 = 100 - 25 = 75$

$OC = \sqrt{75} = 5\sqrt{3}$ cm

$OC = 5 \times 1.7 = 8.5$ cm

Coordinates of third vertex C = (0, –5.5)

29. (a)



In circle with centre O

$\angle TPO = 90^\circ$

(Angle between tangent and radius at point of contact is 90°)

$\angle TQO = 90^\circ$

(Angle between tangent and radius at point of contact is 90°)

In quadrilateral POQT

$\angle POQ + \angle OQT + \angle QTP + \angle TPO = 360^\circ$

(Angle sum property of quadrilateral)

$\angle POQ + 90^\circ + \angle QTP + 90^\circ = 360^\circ$

$\angle POQ + \angle QTP = 360^\circ - 90^\circ - 90^\circ$

$\angle POQ + \angle QTP = 180^\circ$ ...(1)

In ΔPOR

$\angle POQ + \angle OPQ + \angle OQP = 180^\circ$

(Angle sum property of triangle)

$\angle OPQ = \angle OQP$

(Angle opposite to equal radii)

$\angle POQ + 2\angle OPQ = 180^\circ$ ....(2)

From (1) and (2)

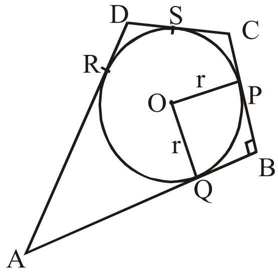
$\angle POQ + \angle PTQ = \angle POQ + 2\angle OPQ$

$\angle PTQ = 2\angle OPQ$

Hence proved

OR

(b)



In quadrilateral ABCD

$\angle B = 90^\circ$

AD = 17 cm (given)

DS = 3 cm (given)

DS = DR

(Length of tangent from external point to circle are equal)

So, DR = 3 cm

Now, AR = AD – DR = 17 – 3 = 14 cm

AR = AQ

(Length of tangent from external point to circle are equal)

BQ = AB – AQ

BQ = 20 – 14 = 6 cm

In quadrilateral BQOP

$\angle POQ + \angle BQO + \angle BPO + \angle PBQ = 360^\circ$

(Angle sum property)

$\angle POQ + 90^\circ + 90^\circ + 90^\circ = 360^\circ$

$\angle POQ = 360^\circ - 90^\circ - 90^\circ - 90^\circ$

$\angle POQ = 90^\circ$

Also OP = OQ = r(Radii of circle)

Hence, PBQO is a square

BQ = r = 6 cm

30. $\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\dfrac{1+\sin\theta}{\cos\theta}$

$$\text{LHS} = \frac{\tan\theta+\sec\theta-(\sec^2\theta-\tan^2\theta)}{(\tan\theta-\sec\theta+1)}$$

$$= \frac{(\tan\theta+\sec\theta)-(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{(\tan\theta-\sec\theta+1)}$$

$$= \frac{(\sec\theta+\tan\theta)[1-\sec\theta+\tan\theta]}{[\tan\theta-\sec\theta+1]}$$

$= (\sec\theta + \tan\theta)$

$$= \frac{1}{\cos\theta}+\frac{\sin\theta}{\cos\theta}$$

$$= \left(\frac{1+\sin\theta}{\cos\theta}\right) = \text{RHS}$$

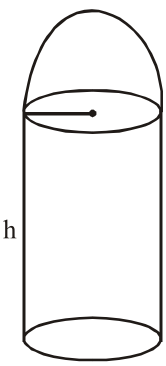
Hence proved

31. (a) $r = \dfrac{1}{2}\times h = \dfrac{h}{2}$

Volume of air in room

= Volume of Hemisphere

+ Volume of Cylinder


h

$$\frac{1408}{21}=\frac{2}{3}\pi r^3+\pi r^2 h$$

$$= \frac{2}{3}\pi\times\left(\frac{h}{2}\right)^3+\pi\left(\frac{h}{2}\right)^2 h$$

$$\frac{1408}{21}=\frac{2}{3}\pi\times\frac{h^3}{8}+\pi\frac{h^3}{4}$$

$$= \pi\frac{h^2}{4}\left\{\frac{2}{3}\times\frac{h}{2}+h\right\}$$

$$= \pi\frac{h^2}{4}\times\frac{4h}{3}$$

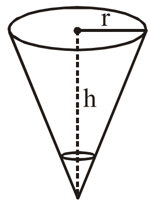
$$\frac{1408}{21}=\frac{22}{7}\times\frac{4}{4}\times\frac{h^3}{3}$$

$$h^3=\frac{1408}{22}=64$$

h = 4 m

OR

(b)



$$= \frac{1}{6}\times\left\{\frac{1}{3}\pi r^2 h\right\}$$

r = 3 cm

h = 12 cm

Required volume of icecream

$$= \frac{2}{3}\pi r^3+\left\{\frac{\pi r^2 h}{3}-\frac{1}{6}\times\frac{\pi r^2 h}{3}\right\}$$

$$= \frac{2}{3}\pi r^3+\frac{\pi r^2 h}{3}\left\{1-\frac{1}{6}\right\}$$

$$= \frac{2}{3}\pi r^3+\frac{5}{6}\times\frac{\pi r^2 h}{3}$$

$$= \frac{1}{3}\pi r^2\left\{2r+\frac{5}{6}\times h\right\}$$

$$= \frac{1}{3}\times\frac{22}{7}\times 9\left\{2\times 3+\frac{5}{6}\times 12\right\}$$

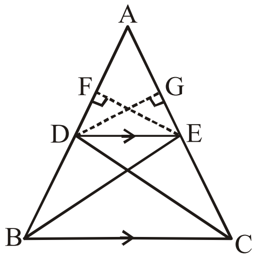
$$= \frac{1}{3}\times\frac{22}{7}\times 9(6+10)$$

$$= \frac{66}{7}\times 16\ \text{cm}^3=\frac{1056}{7}$$

$= 150.85\ \text{cm}^3$

SECTION-D

32.



Given : In ΔABC, $DE \parallel BC$

To prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Proof : Construction Join D to C and B to E

Draw $DG \perp AC$, $EF \perp AB$

Area of $\Delta ADE = \frac{1}{2} \times AD \times EF$ $\{\frac{1}{2} \times \text{Base} \times \text{Height}\}$

Area of $\Delta BDE = \frac{1}{2} \times BD \times EF$

Area of $\Delta ADE = \frac{1}{2} \times AE \times DG$

Area of $\Delta DEC = \frac{1}{2} \times EC \times DG$

$$\frac{Ar\,\Delta ADE}{Ar\,\Delta BDE} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times BD \times EF} = \frac{AD}{BD} \quad ...(1)$$

$$\frac{Ar\,\Delta ADE}{Ar\,\Delta DEC} = \frac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG} = \frac{AE}{EC} \quad ...(2)$$

We known that triangles on same base and between same parallels are equal in area.

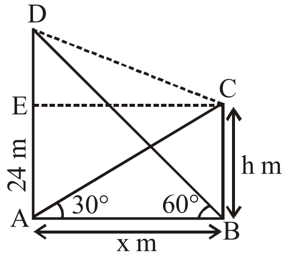
$Ar(\Delta BDE) = Ar(\Delta DEC)$

Hence from (1) and (2)

$$\frac{AD}{BD} = \frac{AE}{EC}$$

Hence proved

33. (a) In ΔABD



$\tan 60° = \frac{AD}{AB}$

$\sqrt{3} = \frac{24}{x}$

$x = \frac{24}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$x = \frac{24\sqrt{3}}{3} = 8\sqrt{3}\text{ m}$

In ΔABC

$\tan 30° = \frac{BC}{AB}$

$\frac{1}{\sqrt{3}} = \frac{h}{8\sqrt{3}}$

$h = \frac{8\sqrt{3}}{\sqrt{3}} = 8\text{ m}$

Height of other tower = 8 m

In ΔCDE

$DE = 24 - 8 = 16$ m

$CE = 8\sqrt{3}\text{ m}$

$DE^2 + CE^2 = DC^2$

$(8\sqrt{3})^2 + (8)^2 = DC^2$

$64 \times 3 + 64 = DC^2$

$64 \times (3 + 1) = DC^2$

$64 \times 4 = DC^2$

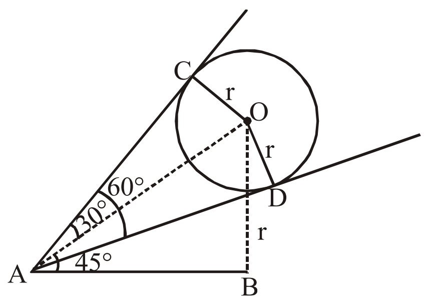
$16 \times 16 = DC^2$

$DC = 16$ m

Length of wire attached to the tops of both towers = 16 m

OR

(b)



In ΔOCA and ΔODA

OC = OD (Radii of same circle)

AO = AO (Common)

AC = AD (Length of tangent from outside point circle are equal)

By SSS property

$\Delta OCA \cong \Delta ODA$

So, $\angle OAD = \angle OAC = 30°$ (Half of 60°)

So, ΔABO

$\sin 45° = \frac{h}{OA}$ (OB = h (let))

$\frac{1}{\sqrt{2}} = \frac{h}{OA}$

$\sqrt{2}h = OA$ .....(1)

In ΔOAD

$\sin 30° = \frac{r}{OA}$

$OA = \frac{r}{\sin 30°} = \frac{r}{\left(\frac{1}{2}\right)}$

OA = 2r .....(2)

from (1) and (2)

$\sqrt{2}h = 2r$

$h = \sqrt{2}r$

Height of centre of Balloon is $\sqrt{2}$ times its radius.

Hence proved

34. In ΔAOB

$\Rightarrow$ OA = OB (Radii of same circle)

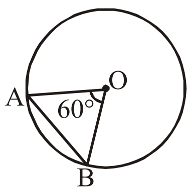
$\angle OAB = \angle OBA$

Let $\angle OAB = \angle OBA = x$

So, $x + x + 60° = 180°$

$2x = 120°$

$\Rightarrow x = 60°$



ΔOAB is an equilateral Δ

$\angle AOB = 60°$

Area of minor segment AB =

Area of sector AOB – Area of ΔAOB

$= \frac{\theta}{360°} \times \pi r^2 - \frac{\sqrt{3}}{4} \times (14)^2$

$= \frac{60°}{360°} \times \frac{22}{7} \times (14)^2 - \frac{\sqrt{3}}{4} \times (14)^2$

= 102.66 – 84.86

= 17.8 cm^2

Area of major segment = Area of circle – Area of minor segment.

$= \pi r^2 - \frac{\theta}{360°} \times \pi r^2$

$= \pi r^2 \left(1 - \frac{60°}{360°}\right)$

$= \frac{22}{7} \times \frac{5}{6} \times 14 \times 14$

= 513.33 cm^2

35. (a) Let first term of AP = a

and common difference = d

According to problem

$\frac{a_{11}}{a_{17}} = \frac{a+10d}{a+16d} = \frac{3}{4}$

4(a + 10d) = 3(a + 16d)

4a + 40d = 3a + 48d

4a – 3a = 48d – 40d

a = 8d ....(1)

Required ratio $= \frac{a_5}{a_{21}} = \frac{a+4d}{a+20d}$

$= \frac{8d+4d}{8d+20d}$

$= \frac{12d}{28d} = \frac{63}{14} = \frac{3}{7}$

Ratio of sum of first 5 forms to sum of first 21 terms.

$$\frac{S_5}{S_{21}} = \frac{\frac{5}{2}[2a+4d]}{\frac{21}{2}[2a+20d]}$$

$= \frac{5}{21} \times \frac{[2(8d)+4d]}{[2(8d)+20d]}$

$= \frac{5}{21} \times \frac{20d}{36d}$

$= \frac{5}{21} \times \frac{5}{9}$

$= \frac{25}{189}$

OR

(b) Total number of wooden logs = 250

a = number of wooden logs in bottom row = 22

Similarly

$a_2 = 21$

$a_3 = 20$

Number of wooden logs in consecutive rows is

22, 21, 20, ... n terms

a = 22

d = 21 – 22 = –1

$S_n = 250$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2(22) + (n-1)(-1)] = 250$

$\frac{n}{2}[44 - n + 1] = 250$

$\frac{n}{2}(45 - n) = 250$

$45n - n^2 = 500$

$n^2 - 45n + 500 = 0$

$n^2 - 25n - 20n + 500 = 0$

$n(n-25) - 20(n-25) = 0$

$(n-25)(n-20) = 0$

n = 20, n = 25

$a_n = a_{20} = a + 19d$

$= 22 + 19(-1)$

$= 22 - 19$

$= 3$

There are 3 wooden logs in last 20th row such that total number of wooden logs is 250.

SECTION-E

36. (i) Length of photo = 18 cm

Breadth of photo = 12 cm

According to problem

$(18 + x)(12 + x) = (18 \times 12) \times 2$

$18 \times 12 + 12x + 18x + x^2 = 18 \times 12 \times 2$

$x^2 + 30x = 18 \times 12$

$x^2 + 30x - 216 = 0$

(ii) Required standard form of quadratic equation is

$1.x^2 + 30.x + (-216) = 0$

(iii) $x^2 + 30x - 216 = 0$

$x^2 + 36x - 6x - 216 = 0$

$x(x + 36) - 6(x + 36) = 0$

$(x - 6)(x + 36) = 0$

x = 6, x = –36

New length = 18 + 6 = 24 cm

Breadth = 12 + 6 = 18 cm

OR

$(18 + x)(12 + x) = 220$

$216 + 30x + x^2 = 220$

$x^2 + 30x - 4 = 0$

$D = b^2 - 4ac = 900 + 16 = 916$

As D is not perfect square

$\Rightarrow$ For no rational value of x area is 220 cm^2.

37.

Rain fall	Number of sub.division	c.f.
200–400	2	2
400–600	4	6
600–800	7	13
800–1000	4	17
1000–1200	2	19
1200–1400	3	22
1400–1600	1	23
1600–1800	1	24

(i) Highest frequency is 7 of class interval 600 – 800 so the modal class is 600 – 800

(ii) $\frac{N}{2} = \frac{24}{2} = 12$

Median class = 600 – 800

$$\text{Median} = \ell + \frac{\frac{N}{2} - c.f.}{f} \times h$$

$$= 600 + \frac{12-6}{7} \times 200$$

$$= 600 + \frac{6}{7} \times 200$$

$$= 600 + \frac{1200}{7}$$

= Median = 771.42

OR

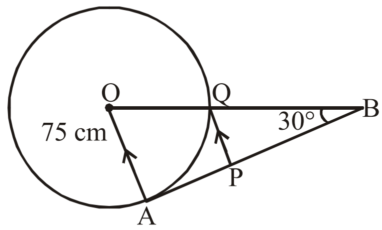
Rain fall	Number of sub.division	x_i	f_ix_i
200–400	2	300	600
400–600	4	500	2000
600–800	7	700	4900
800–1000	4	900	3600
1000–1200	2	1100	2200
1200–1400	3	1300	3900
1400–1600	1	1500	1500
1600–1800	1	1700	1700
	24	$\Sigma f_ix_i = 20,400$	

$$\bar{x} = \frac{\Sigma f_ix_i}{\Sigma f_i} = \frac{20,400}{24}$$

$$\bar{x} = 850$$

(iii) Number of sub-division with good rainfall more than 1000 mm is 7(2 + 3 + 1 + 1)

38. (a)



In ΔAOB, $\angle ABO = 30°$ (given)

OA = 75 cm (given)

$\angle OAB = 90°$

(Angle between tangent and radius)

$$\tan 30° = \frac{OA}{AB}$$

$$\frac{1}{\sqrt{3}} = \frac{75}{AB}$$

$$AB = 75\sqrt{3}\text{ cm}$$

(b) In ΔOAB

$$\sin 30° = \frac{OA}{OB} = \frac{75}{OB}$$

OB = 150 cm

(c) OQ = 75 cm (Radius of circle)

In ΔAOB

$$\frac{75}{BQ + 75} = \sin 30°$$

$$75 = \frac{1}{2} \times (BQ + 75)$$

150 = BQ + 75

BQ = 150 – 75

BQ = 75 cm

In ΔQPB & ΔOAB

$90° = \angle OAB = \angle QPB$

(corresponding angles)

$\angle OBA = \angle QBP$ (common)

$\Delta OAB \sim \Delta QPB$ (by AA similarity)

$$\frac{PB}{AB} = \frac{QB}{OB}$$

$$\frac{PB}{75\sqrt{3}} = \frac{75}{150}$$

$$PB = \frac{75 \times 75\sqrt{3}}{150}$$

$$PB = 37.5\sqrt{3}$$

$$AP = 75\sqrt{3} - 37.5\sqrt{3}$$

$$AP = 37.5\sqrt{3}\,\text{cm}$$

OR

$$\frac{PQ}{75} = \frac{QB}{OB}$$

$$\frac{PQ}{75} = \frac{75}{150}$$

$$PQ = \frac{75 \times 75}{150}$$

PQ = 37.5 cm

MATHEMATICS SAMPLE PAPER # 1

ANSWER AND SOLUTIONS

SECTION-A

1. Option (1)

165

2. Option (3)

20

3. Option (1)

All real values except 10

4. Option (4)

Not defined

5. Option (2)

12

6. Option (3)

$\frac{11}{36}$

7. Option (4)

IV quadrant

8. Option (3)

4

9. Option (1)

–12

10. Option (2)

$\pi r(\ell + 2h + r)$

11. Option (4)

4

12. Option (1)

14, 38

13. Option (3)

$\frac{3}{11}$

14. Option (2)

5

15. Option (3)

12 cm

16. Option (2)

2r

17. Option (2)

2 : 3

18. Option (3)

±3

19. Option (2)

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

20. Option (3)

Assertion (A) is true but Reason (R) is false.

SECTION-B

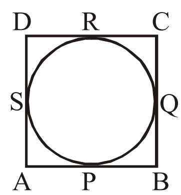
21. 110, 120, 130, ..., 990

$a_n = 990$

$\Rightarrow 110 + (n - 1) \times 10 = 990$

$\therefore n = 89$

22.



AP = AS, BP = BQ, CR = CQ and DR = DS

$\Rightarrow$ AP + BP + CR + DR = AS + BQ + CQ + DS

$\Rightarrow$ AB + CD = AD + CB

But AB = CD and AD = CB

$\therefore$ AB = AD

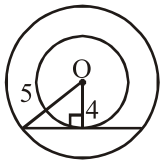
Hence, ABCD is a square.

OR

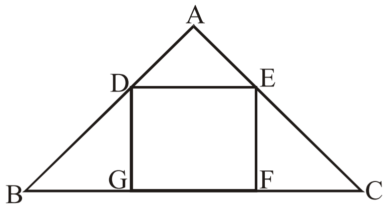
Length of tangent $= 2\times\sqrt{5^2-4^2}$

$= 2\times 3$ cm

$= 6$ cm



23.



$\Delta ADE \sim \Delta GBD$ and $\Delta ADE \sim \Delta FEC$

$\Rightarrow \Delta GBD \sim \Delta FEC$ (AA Criterion)

$\Rightarrow \frac{GD}{FC} = \frac{GB}{FE} \Rightarrow GD \times FE = GB \times FC$

or $FG^2 = BG \times FC$ $[\because GD = FE = FG]$

Hence proved

24. Capacity of first glass $= \pi r^2 H - \frac{2}{3}\pi r^3$

$= \pi \times 9\ (10 - 2) = 72\pi cm^3$

Capacity of second glass $= \pi r^2 H - \frac{1}{3}\pi r^2 h$

$= \pi \times 3 \times 3\ (10 - 0.5) = 85.5\pi cm^3$

$\therefore$ Suresh got more quantity of juice.

Extra amount $= 13.5\ \pi cm^3$

25. For Jayanti,

Favourable outcome is (6, 6) i.e, 1

Probability (getting the number 36) $= \frac{1}{36}$

For Pihu,

Favourable outcome is 6 i.e, 1

Probability (getting the number 36) $= \frac{1}{6}$

$\therefore$ Pihu has the better chance.

OR

Total number of integers = 29

(i) Prob. (Prime number) $= \frac{6}{29}$

(ii) Prob. (Number divisible by 7) $= \frac{4}{29}$

SECTION-C

26. Let us assume to the contrary, that $2\sqrt{5}-3$ is a rational number

$\therefore\ 2\sqrt{5} - 3 = \frac{p}{q}$, where p and q are integers and coprime and $q \neq 0$

$\Rightarrow \sqrt{5} = \frac{p+3q}{2q}$ ..... (1)

Since p and q are integers $\therefore \frac{p+3q}{2q}$ is a rational number.

$\therefore\ \sqrt{5}$ is a rational number which is a contradiction as $\sqrt{5}$ is an irrational number.

Hence our assumption is wrong and hence $2\sqrt{5}-3$ is an irrational number.

OR

$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$

$180 = 2 \times 2 \times 3 \times 3 \times 5$

HCF $= 2 \times 2 \times 3 \times 3 = 36$

$13m - 16 = 36$

$13m = 52$

$m = 4$

27. $x + y = 7$ and $2(x - y) + x + y + 5 + 5 = 27$

$\therefore\ x + y = 7$ and $3x - y = 17$

Solving, we get, $x = 6$ and $y = 1$

28. (i) A(1, 7), B(4, 2), C(–4, 4)

Distance travelled by Seema $= \sqrt{34}$ units

Distance travelled by Aditya $= \sqrt{68}$ units

$\therefore$ Aditya travels more distance

(ii) Coordinate of D are

$\left(\frac{1+4}{2}, \frac{7+2}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$

29. $\sin\theta + \cos\theta = \sqrt{3} \Rightarrow (\sin\theta + \cos\theta)^2 = 3$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 3 \Rightarrow \sin\theta\cos\theta = 1$

$\therefore \tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = 1$

Hence proved

30. Required Area = Area of triangle - Area of 3 sectors

Area of Triangle = $\dfrac{1}{2} \times 24 \times 7 = 84\ m^2$

Area of three sectors

$= \dfrac{\pi r^2}{360°} \times$ (sum of three angles of triangle)

$= \dfrac{22 \times 7 \times 7 \times 180°}{7 \times 2 \times 2 \times 360°} = \dfrac{77}{4}$ or $19.25\ m^2$

$\therefore$ Required Area = $\dfrac{259}{4}$ or $64.75\ m^2$

OR

Quantity of water flowing through pipe in 1 hour

$= \pi \times \dfrac{7}{100} \times \dfrac{7}{100} \times 15000 m^3$

Required time

$= \left(50 \times 44 \times \dfrac{21}{100}\right) \div (\pi \times \dfrac{7}{100} \times \dfrac{7}{100} \times 15000)$

= 2 hours

31. LHS : $\dfrac{\dfrac{\sin^3\theta}{\cos^3\theta}}{1 + \dfrac{\sin^2\theta}{\cos^2\theta}} + \dfrac{\dfrac{\cos^3\theta}{\sin^3\theta}}{1 + \dfrac{\cos^2\theta}{\sin^2\theta}}$

$= \dfrac{\dfrac{\sin^3\theta}{\cos^3\theta}}{\dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}} + \dfrac{\dfrac{\cos^3\theta}{\sin^3\theta}}{\dfrac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}}$

$= \dfrac{\sin^3\theta}{\cos\theta} + \dfrac{\cos^3\theta}{\sin\theta}$

$= \dfrac{\sin^4\theta + \cos^4\theta}{\cos\theta\sin\theta}$

$= \dfrac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$

$= \dfrac{1 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$

$= \dfrac{1}{\cos\theta\sin\theta} - \dfrac{2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$

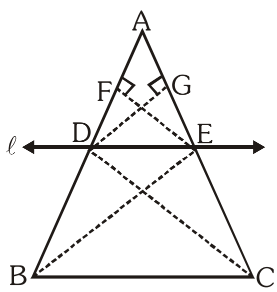
$= \sec\theta\ \mathrm{cosec}\theta - 2\sin\theta\cos\theta$

= RHS

SECTION-D

32. **Given :** A ΔABC in which line ℓ parallel to BC ($DE \| BC$) intersecting AB at D and AC at E.

To prove : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$



Construction : Join D to C and E to B. Through E draw EF perpendicular to AB i.e., $EF \perp AB$ and through D draw $DG \perp AC$.

Proof :

Area of $(\Delta ADE) = \dfrac{1}{2}(AD \times EF)$ ...(1)

(Area of $\Delta = \dfrac{1}{2}$ base $\times$ altitude)

Area of $(\Delta BDE) = \dfrac{1}{2}(BD \times EF)$ ...(2)

Dividing (1) by (2)

$$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta BDE)} = \frac{\frac{1}{2}AD \times EF}{\frac{1}{2}BD \times EF} = \frac{AD}{DB} \quad ...(3)$$

Similarly, $$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta CDE)} = \frac{\frac{1}{2}AE \times DG}{\frac{1}{2}EC \times DG} = \frac{AE}{EC}$$

$$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta CDE)} = \frac{AE}{EC} \quad ...(4)$$

Area (ΔBDE) = Area (ΔCDE) ...(5)

[Δs BDE and CDE are on the same base DE and between the same parallel lines DE and BC.]

From (4) and (5)

$$\frac{\text{Area}(\Delta ADE)}{\text{Area}(\Delta BDE)} = \frac{AE}{EC} \quad ...(6)$$

From (3) and (6)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Hence proved.

33. Let the original speed of the train be x km/h

$$\therefore \quad \frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$$

$\Rightarrow \quad x^2 + 5x - 2250 = 0$

$\Rightarrow \quad (x + 50)(x - 45) = 0$

$\therefore \quad x = 45$

Hence original speed of the train = 45km/h

OR

$$\frac{1}{x} - \frac{1}{x-2} = 3$$

$$\frac{x-2-x}{x(x-2)} = \frac{3}{1}$$

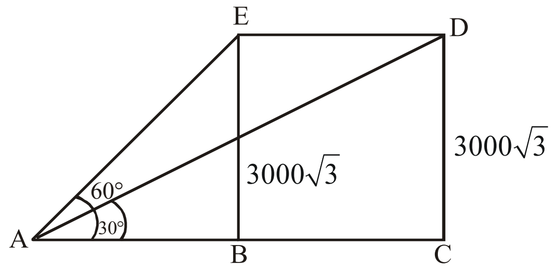
$3x^2 - 6x = -2$

$3x^2 - 6x + 2 = 0$

$$x = \frac{6 \pm \sqrt{12}}{6}$$

$$= \frac{3+\sqrt{3}}{3}, \frac{3-\sqrt{3}}{3}$$

34.



In ΔABE, $\frac{BE}{AB} = \tan 60°$

$\Rightarrow$ AB = 3000 m

In ΔDAC, $\frac{DC}{AC} = \tan 30°$

$\Rightarrow$ AC = 9000 m

BC = AC – AB = 6000m

$\therefore$ Speed of aeroplane = $\frac{6000}{30}$ m/s = 200m/s

= 720 km/hr

35.

Daily Wages (in Rs.)	Number of Workers(f_i)	x_i	$u_i = \frac{x_i - a}{h}$	f_iu_i
100-120	10	110	– 3	– 30
120-140	15	130	– 2	– 30
140-160	20	150	– 1	– 20
160-180	22	170	0	0
180-200	18	190	1	18
200-220	12	210	2	24
220-240	13	230	3	39
Total	110			1

Here, a = 170

Mean daily wages = $170 + \frac{1}{110} \times 20 = ₹170.19$ (approx.)

Mode = $160 + \frac{22-20}{44-20-18} \times 20 = ₹166.67$ (approx.)

OR

Re-writing the distribution in the form of the grouped distribution with each class interval as 10 and taking assumed mean to be 55, we get the following table.

Class	Mid – value $\left(x_i = \frac{\ell + u}{2}\right)$	$d_i = x_i - A$ (A = 55)	$u_i = \frac{d_i}{h}$	Number of students (f_i)	f_iu_i
0 – 10	5	–50	–5	12	–60
10 – 20	15	–40	–4	10	–40
20 – 30	25	–30	–3	13	–39
30 – 40	35	–20	–2	15	–30
40 – 50	45	–10	–1	20	–20
50 – 60	55 = A	0	0	16	0
60 – 70	65	10	1	11	11
70 – 80	75	20	2	7	14
80 – 90	85	30	3	5	15
90 – 100	95	40	4	6	24

Mean = $A + \frac{f_iu_i}{f_i} \times h = 55 + \frac{-125}{115} \times 10$

= 44.13 (approx)

SECTION-E

36. (i) Since each row is increasing by 10 seats, so it is an AP with first term a = 30, and common difference d = 10.

So number of seats in 10^{th} row

$= a_{10}$

= a + 9d

= 30 + 9 × 10 = 120

(ii) $S_n = \frac{n}{2}[2 \times 30 + (n-1)10]$

$1500 = \frac{n}{2}[2 \times 30 + (n-1)10]$

$3000 = 50n + 10n^2$

$n^2 + 5n - 300 = 0$

$n^2 + 20n - 15n - 300 = 0$

$(n + 20)(n - 15) = 0$

Rejecting the negative value, n = 15

OR

Number of seats already put up to the 10^{th} row = S_{10}

$S_{10} = \frac{10}{2}\{(2 \times 30 + (10-1)10)\}$

= 5(60 + 90) = 750

So, the number of seats still required to be put are 1500 – 750 = 750

(iii) If number of rows = 17

then the middle row is the 9^{th} row

$a_9 = a + 8d = 30 + 80 = 110$ seats

37. (i) Let AD be x cm, then DB = (12 – x) cm

∵ AD = AF, CF = CE, DB = BE

[tangents to a circle from an external point]

∴ AF = xcm,

then CF = (10 – x)cm

BE = (12 – x)cm,

then CE = 8 – (12 – x) = (x – 4) cm

Now CF = CE

10 – x = x – 4

2x = 14

⇒ x = 7 ....(1)

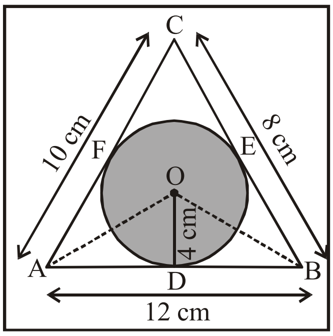
Hence, AD = 7 cm

Since, ∵ BE = (12 – x)cm = (12 – 7) cm [from (1)]

BE = 5 cm

(ii) Radius, OD = 4 cm

and AB = 12 cm



Then, area of ΔOAB

$= \frac{1}{2} \times OD \times AB$

$= \frac{1}{2} \times 4 \times 12$

$= 24\ cm^2$

(iii) Perimeter of $\Delta ABC = AB + BC + CA$

$= (12 + 8 + 10)$ cm

= 30 cm

OR

Since, 100 cm cost = Rs.1500

So, 30 cm cost $= \frac{1500 \times 30}{100}$ = Rs.450

38. (i) For cuboid

$\ell = 15$ cm, b = 10 cm and h = 3.5 cm

Volume of the cuboid $= \ell \times b \times h$

$= 15 \times 10 \times 3.5$

$= 525\ cm^3$

(ii) For conical depression :

r = 0.5 cm,

h = 1.4 cm

Volume of conical depression

$= \frac{1}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 1.4$

$= \frac{11}{30}\ cm^3$

(iii) Volume of four conical depressions

$= 4 \times \frac{11}{30} = 1.47\,cm^3$

OR

Volume of the wood in the entire stand

= Volume of cuboid – Volume of 4 conical depressions

$= 525 - 1.47$

$= 523.53\ cm^3$

MATHEMATICS SAMPLE PAPER # 2

ANSWER AND SOLUTIONS

SECTION-A

1. Option (4)
More than 3

2. Option (3)
$$\cos\theta = \frac{\sqrt{b^2 - a^2}}{b}$$

3. Option (2)
$a_n = 3.5$

4. Option (3)
Trigonometric ratios of the angles.

5. Option (2)
7.6

6. Option (3)
10

7. Option (2)
2.1

8. Option (2)
$$\frac{5}{2}$$

9. Option (1)
360 cm^2

10. Option (4)
Median

11. Option (3)
2 and –2

12. Option (4)
4

13. Option (3)
12

14. Option (4)
7000

15. Option (2)
$$\sqrt{34}$$

16. Option (3)
$$\frac{1}{4}$$

17. Option (2)
28

18. Option (2)
$$\frac{BE}{EC}$$

19. Option (4)
Assertion (A) is false but Reason (R) is true.

20. Option (2)
Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

SECTION-B

21. In ΔPAO and ΔQBO,
$\angle A = \angle B = 90°$ (Given)
$\angle POA = \angle QOB$ (Vertically opposite angles)
Since, $\Delta PAO \sim \Delta QBO$, (by AA similarity)

Then, $\frac{OA}{OB} = \frac{PA}{QB}$

or, $\frac{6}{4.5} = \frac{4}{QB}$

or, $QB = \frac{4 \times 4.5}{6}$

$\therefore$ QB = 3 cm

22. Here, the total number of possible outcomes = 5.
(i) Since, there is only one queen
$\therefore$ Favourable number of elementary events = 1

$\therefore$ Probability of getting the card of queen $= \frac{1}{5}$.

(ii) Now, the total number of possible outcomes = 4.
Since, there is only one ace
$\therefore$ Favourable number of elementary events = 1

$\therefore$ Probability of getting an ace card $= \frac{1}{4}$.

23. HCF × LCM = Product of two numbers

$9 \times 360 = 45 \times$ 2nd number

2nd number = 72

OR

Let us assume, to the contrary that $7-\sqrt{5}$ is rational

$7-\sqrt{5} = \frac{p}{q}$, where p & q are co-prime and $q \neq 0$

$\Rightarrow \sqrt{5} = \frac{7q-p}{q}$

$\frac{7q-p}{q}$ is rational = $\sqrt{5}$ is rational which is a contradiction

Hence $7-\sqrt{5}$ is irrational

24. 20th term from the end = $\ell - (n-1)d$

$= 253 - 19 \times 5$

$= 158$

OR

$7a_7 = 11a_{11}$

$\Rightarrow 7(a + 6d) = 11(a + 10d)$

$\Rightarrow 4a + 68d = 0$

$\Rightarrow a + 17d = 0$

$\Rightarrow a_{18} = 0$

25. $x = \frac{6-6}{5} = 0$

$y = \frac{-10+15}{5} = 1$

Hence, coordinates of point P(0, 1)

SECTION-C

26. Let the numerator be x and denominator be y.

$\therefore$ Fraction = $\frac{x}{y}$

Now, according to question,

$\frac{x-1}{y} = \frac{1}{3} \quad \Rightarrow \quad 3x - 3 = y$

$\therefore 3x - y = 3$ ....(i)

and $\frac{x}{y+8} = \frac{1}{4} \quad \Rightarrow \quad 4x = y + 8$

$\therefore 4x - y = 8$ ....(ii)

Now, subtracting equation (ii) from (i), we have

$$\begin{array}{r} 3x - y = 3 \\ 4x - y = 8 \\ - \quad + \quad - \\ \hline -x = -5 \end{array}$$

x = 5

Putting the value of x in equation (i), we have

$3 \times 5 - y = 3 \Rightarrow 15 - y = 3 \Rightarrow 15 - 3 = y$

$\therefore y = 12$

Hence, the required fraction is $\frac{5}{12}$.

OR

Let the speed of car at A be x km/h

And the speed of car at B be y km/h

Case 1 $8x - 8y = 80$

$x - y = 10$

Case 2 $\frac{4}{3}x + \frac{4}{3}y = 80$

$x + y = 60$

On solving x = 35 and y = 25

Hence, speed of cars at A and B are 35 km/h and 25 km/h respectively

27. LHS = $\sin\theta(1 + \tan\theta) + \cos\theta(1 + \cot\theta)$

$= \sin\theta + \sin\theta \cdot \frac{\sin\theta}{\cos\theta} + \cos\theta + \cos\theta\frac{\cos\theta}{\sin\theta}$

$= (\sin\theta + \cos\theta) + \frac{\sin^2\theta}{\cos\theta} + \frac{\cos^2\theta}{\sin\theta}$

$= (\sin\theta + \cos\theta) + \frac{\sin^3\theta + \cos^3\theta}{\sin\theta\cos\theta}$

$= (\sin\theta + \cos\theta)\left[1 + \frac{\sin^2\theta + \cos^2\theta - \sin\theta\cos\theta}{\sin\theta\cos\theta}\right]$

$= (\sin\theta + \cos\theta)\left[1+\dfrac{1}{\sin\theta\cos\theta}-1\right]$

$= (\sin\theta + \cos\theta) \times \dfrac{1}{\sin\theta\cos\theta}$

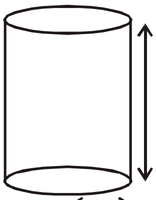
$= \dfrac{1}{\cos\theta}+\dfrac{1}{\sin\theta}$

$= \sec\theta + \operatorname{cosec}\theta$

= RHS

Hence proved

28. Volume of cylindrical bucket = Volume of conical heap of sand.



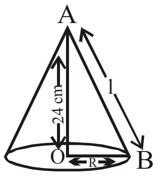
$\pi r^2 h = \dfrac{1}{3}\pi R^2 \times 24$

$\pi \times 18 \times 18 \times 32$

$= \dfrac{1}{3}\pi R^2 \times 24$

$R^2 = \dfrac{18\times18\times32\times3}{24} = \dfrac{18\times18\times32\times3}{24}$

R = 36 cm



In the ΔAOB of conical heap.

$AB^2 = AO^2 + OB^2$

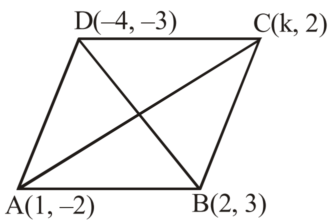
$\ell^2 = 24^2 + 36^2$

$\ell = \sqrt{576+1296}$

$= \sqrt{1872}$

ℓ = 43.27 cm = 43.3 cm

29.



Diagonals of parallelogram bisect each other

$\Rightarrow$ midpoint of AC = midpoint of BD

$\Rightarrow \left(\dfrac{1+k}{2}, \dfrac{-2+2}{2}\right) = \left(\dfrac{-4+2}{2}, \dfrac{-3+3}{2}\right)$

$\Rightarrow \dfrac{1+k}{2} = \dfrac{-2}{2}$

$\Rightarrow k = -3$

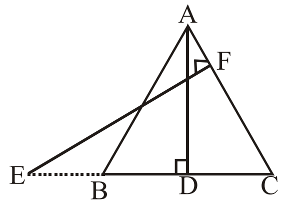
30. 200 – 250 is the modal class

$\text{Mode} = \ell + \dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$= 200 + \dfrac{12-5}{24-5-2} \times 50$

= 200 + 20.59 = ₹220.59

31.



In ΔABD and ΔCEF

AB = AC (Given)

$\Rightarrow \angle ABC = \angle ACB$

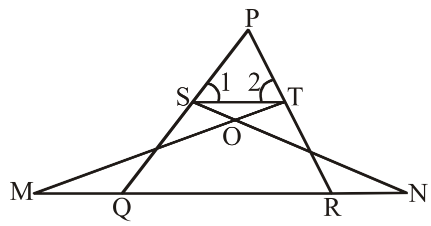
(Equal sides have equal oppposite angles)

$\angle ABD = \angle ECF$

$\angle ADB = \angle EFC$ [Each 90°]

So, $\Delta ABD \sim \Delta ECF$ (AA – Similarity)

OR



$\angle 1 = \angle 2$ (Given)

PT = PS

(Side opposite to equal angles are equal) ....(1)

$\Delta NSQ \cong \angle MTR$

$\angle Q = \angle R$ (by cpct)

PQ = PR

(Side opposite to equal angles are equal) ....(2)

From (1) and (2)

$\dfrac{PT}{PR} = \dfrac{PS}{PQ}$

$\angle P = \angle P$

$\Delta PTS \sim \Delta PRQ$ (by SAS Similarity)

SECTION-D

32. $S_n = \dfrac{n}{2}[2a + (n-1)d]$

$S_{30} = \dfrac{30}{2}[2a + 29d] \Rightarrow S_{30} = 30a + 435d$ ...(i)

$\Rightarrow S_{20} = \dfrac{20}{2}[2a + 19d] \Rightarrow S_{20} = 20a + 190d$

$S_{10} = \frac{10}{2}[2a + 9d] \Rightarrow S_{10} = 10a + 45d$

$3(S_{20} - S_{10}) = 3[20a + 190d - 10a - 45d]$

$= 3[10a + 145d] = 30a + 435d = S_{30}$

[From (i)]

Hence, $S_{30} = 3(S_{20} - S_{10})$ Hence proved.

OR

Sum of first seven terms,

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_7 = \frac{7}{2}[2a + (7-1)d] = \frac{7}{2}[2a + 6d]$

$\Rightarrow 63 = 7a + 21d$

$\Rightarrow a = \frac{63 - 21d}{7}$ ... (1)

$\Rightarrow S_{14} = \frac{14}{2}[2a + 13d]$

$\Rightarrow S_{14} = 7[2a + 13d] = 14a + 91d$

But ATQ,

$S_{1-7} + S_{8-14} = S_{14}$

$63 + 161 = 14a + 91d$

$\Rightarrow 224 = 14a + 91d$

$2a + 13d = 32$

$2\left(\frac{63 - 21d}{7}\right) + 13d = 32$ (from 1)

$\Rightarrow 126 - 42d + 91d = 224$

$\Rightarrow 49d = 98$

$\Rightarrow d = 2$

$\Rightarrow a = \frac{63 - 21 \times 2}{7} = \frac{63 - 42}{7} = 3$

$\Rightarrow a_{28} = a + 27d = 3 + 27 \times 2$

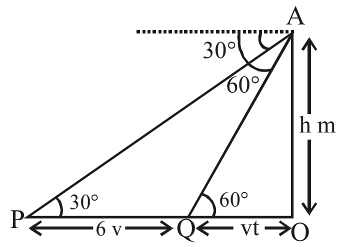
$\Rightarrow a_{28} = 3 + 54 = 57$

33. Let OA be the tower of height h, and P be the initial position of the car when the angle of depression is 30°.

After 6 seconds, the car reaches to Q such that the angle of depression at Q is 60°. Let the speed of the car be v metre per second. Then,

PQ = 6v (∵ Distance = speed × time)

and let the car take t seconds to reach the tower OA from Q (Figure). Then OQ = vt metres.



Now, in ΔAQO we have

$\tan 60° = \frac{OA}{QO}$

$\Rightarrow \sqrt{3} = \frac{h}{vt} \Rightarrow h = \sqrt{3}\ vt$ ....(i)

Now, in ΔAPO, we have

$\tan 30° = \frac{OA}{PO}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{6v + vt} \Rightarrow \sqrt{3}h = 6v + vt$ ....(ii)

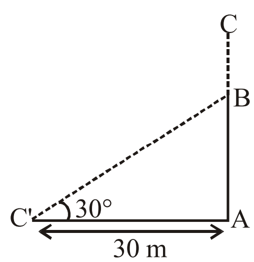
Now, substituting the value of h from (i) and into (ii), we have

$\sqrt{3} \times \sqrt{3}\ vt = 6v + vt$

$\Rightarrow 3vt = 6v + vt \Rightarrow 2vt = 6v \Rightarrow t = \frac{6v}{2v} = 3$

Hence, the car will reach the tower from Q in 3 seconds.

OR



Let AC be the tree and BC' be the broken part.

In $\Delta ABC'$

$\tan 30^\circ = \frac{AB}{30}$

$\frac{1}{\sqrt{3}} = \frac{AB}{30}$

$\frac{30}{\sqrt{3}} = AB$

$AB = 10\sqrt{3}$ m

Also $\cos 30^\circ = \frac{AC'}{BC'}$

$\frac{\sqrt{3}}{2} = \frac{30}{BC'}$

$BC = \frac{60}{\sqrt{3}}$

$BC = 20\sqrt{3}$ m

Total height = AB + BC

$= 10\sqrt{3} + 20\sqrt{3}$

$= 30\sqrt{3}$ m

34. In ΔAPE and ΔBPF,

$\angle APE = \angle BPF$ [Vertically opposite angles]

$\angle AEP = \angle BFP$ [Alternate angles]

By AA similarity, $\Delta APE \sim \Delta BPF$

Thus, $\frac{AP}{BP} = \frac{PE}{PF} = \frac{AE}{BF}$ ....(1)

In ΔCPE and ΔDPF,

$\angle CPE = \angle DPF$ [Vertically opposite angles]

$\angle CEP = \angle DFP$ [Alternate angles]

By AA similarity, $\Delta CPE \sim \Delta DPF$

Thus, $\frac{CP}{DP} = \frac{PE}{PF} = \frac{CE}{DF}$ ....(2)

In ΔAPC and ΔBPD,

$\angle APC = \angle BPD$ [Vertically opposite angles]

$\angle ACP = \angle BDP$ [Alternate angles]

By AA similarity, $\Delta APC \sim \Delta BPD$

Thus, $\frac{AP}{BP} = \frac{PC}{PD} = \frac{AC}{BD}$ ....(3)

From equations (1), (2) and (3), we get

$\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$

Hence proved

35.

Class Interval	Frequency	cf
0 – 100	2	2
100 – 200	5	7
200 – 300	x	7 + x
300 – 400	12	19 + x
400 – 500	17	36 + x
500 – 600	20	56 + x
600 – 700	y	56 + x + y
700 – 800	9	65 + x + y
800 – 900	7	72 + x + y
900 – 1000	4	76 + x + y

N = 100

$\Rightarrow 76 + x + y = 100$

$\Rightarrow x + y = 24$ ...(i)

Median = 525

$\Rightarrow$ 500 – 600 is median class

$$\text{Median} = \ell + \frac{\frac{n}{2} - cf}{f} \times h$$

$$\Rightarrow 500 + \left(\frac{50 - 36 - x}{20}\right) \times 100 = 525$$

$\Rightarrow (14 - x) \times 5 = 25$

$\Rightarrow x = 9$

$\Rightarrow$ from (1), y = 15

SECTION-D

36. (i) Let the fixed charge for two days be Rs.x and additional charge be Rs.y per day.

As Radhika has taken book for 4 days.

It means that Radhika will pay fixed charge for first two days and pays additional charges for next two days.

$x + 2y = 16$

(ii) As the fixed charge for two days be Rs.x and additional charge be Rs.y per day

It means that Amruta will pay fixed charge for first two days and pays additional charges for next four days.

$x + 4y = 22$

(iii) $x + 4y = 22$ .....(i)

$x + 2y = 16$ .....(ii)

On solving (i) and (ii)

Therefore, additional charges is y = Rs.3

OR

For two more days price charged will be

$2y = 2 \times 3 = 6$

Total money paid by Amruta and Radhika is 22 + 16 + 6 + 6 = Rs.50

37. (i) Number of rose plants = 135

Number of marigold plants = 225

The maximum number of columns in which they can be planted

= HCF of 135 and 225

$\therefore$ Prime factors of $135 = 3 \times 3 \times 3 \times 5$

and $225 = 3 \times 3 \times 5 \times 5$

$\therefore$ Maximum number of columns = HCF (135, 225) $= 3 \times 3 \times 5 = 45$

OR

Total number of plants 135 + 225 = 360 plants

(ii) We have proved that the maximum number of columns = 45

So, prime factors of $45 = 3 \times 3 \times 5$

$= 3^2 \times 5^1$

$\therefore$ Sum of exponents = 2 + 1 = 3.

(iii) Number of rows of Rose plants $= \frac{135}{45} = 3$

Number of rows of marigold plants $= \frac{225}{45} = 5$

Total number of rows = 3 + 5 = 8

38. (i) Area of grass field $= 15 \times 15 = 225 \text{ m}^2$

(ii) Area of field horse can graze $= \frac{1}{4}\pi 5^2$

$= \frac{1}{4} \times \frac{22}{7} \times 25$

$= 19.64 \text{ m}^2$.

(iii) If rope was 10 m of grazing field

$= \frac{1}{4} \times \frac{22}{7} \times 100 = 78.57 \text{ m}^2$

OR

Increase in area $= 78.57 - 19.64 = 58.93 \text{ m}^2$

MATHEMATICS — SAMPLE PAPER # 3

ANSWER AND SOLUTIONS

SECTION-A

1. Option (2)
-1

2. Option (1)
(3, 1)

3. Option (2)
$k \le 4$

4. Option (4)
$4\sqrt{2}$ cm

5. Option (1)
$60°$

6. Option (4)
9 units

7. Option (1)
7.8

8. Option (2)
162

9. Option (1)
$-\frac{9}{4}$

10. Option (1)
0

11. Option (4)
3

12. Option (3)
25

13. Option (4)
16.8 cm

14. Option (3)
$\frac{5}{4}$

15. Option (3)
4

16. Option (4)
17.5

17. Option (2)
$\tan 30°$

18. Option (1)
$\sqrt{119}$ cm

19. Option (4)
Assertion (A) is false but Reason (R) is true.

20. Option (3)
Assertion (A) is true but Reason (R) is false.

SECTION-B

21. Number divisible by 8 between 200 and 500 are 208, 216, 224, ...496 which forms an A.P.

$\therefore$ First term (a) = 208, common difference (d) = 8

n^{th} term of an A.P. is $a_n = a + (n - 1)d$

$496 = 208 + (n - 1)8$

$\Rightarrow \quad 288 = (n - 1)8$

$\Rightarrow \quad n - 1 = 36$

$\Rightarrow \quad n = 37$

OR

Here, $a = 16$, $\ell = 128$

$$S_n = \frac{n}{2}(a + \ell)$$

$$= \frac{8}{2}(16 + 128)$$

$$= 4 \times 144$$

$$= 576$$

22. Total possible outcomes = 6 × 6 = 36

Favourable outcomes are {(1, 6), (2, 3), (3, 2), (6, 1)} i.e. 4 in number.

$\therefore$ P(getting the product 6) = $\frac{4}{36} = \frac{1}{9}$

23. If height is 40 cm

circumference of base of cylinder = 22 cm

$$2 \times \frac{22}{7} \times r = 22$$

$$r = \frac{7}{2} \text{ cm}$$

24. Any number which ends in zero must have at least 2 and 5 as prime factors.

$6 = 2 \times 3$

$6^n = (2 \times 3)^n$

$= 2^n \times 3^n$

Hence, prime factor of 6 are 2 and 3

Thus, 6^n can never end with digit 0.

OR

$90 = 2 \times 3^2 \times 5$

$144 = 2^4 \times 3^2$

$HCF = 2 \times 3^2 = 18$

$LCM = 2^4 \times 3^2 \times 5 = 720$

25. Let P(x, y) is equidistant from A(–5, 3) and B(7, 2)

AP = BP

$\Rightarrow \sqrt{((x+5)^2+(y-3)^2)} = \sqrt{((x-7)^2+(y-2)^2)}$

$\Rightarrow x^2 + 10x + 25 + y^2 - 6y + 9$

$= x^2 - 14x + 49 + y^2 - 4y + 4$

$10x - 6y + 34 = -14x - 4y + 53$

$10x + 14x - 6y + 4y = 53 - 34$

$24x - 2y = 19$

$24x - 2y - 19 = 0$

is the required relation.

SECTION-C

26. Radius of the cylinder (r) = 3.5 cm

Height of the cylinder (h) = 10 cm

Curved surface area of cylinder = $2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{35}{10} \times 10 \text{ cm}^2$

$= 220 \text{ cm}^2$

Curved surface area of a hemisphere = $2\pi r^2$

Curved surface area of both hemispheres

$= 2 \times 2\pi r^2 = 4\pi r^2 = 4 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} \text{cm}^2$

$= 154 \text{ cm}^2$

Total surface area of the remaining solid

= (Curved surface area of cylinder + curved surface area of 2 hemispheres)

$= (220 + 154) \text{ cm}^2 = 374 \text{ cm}^2$.

OR

Given : d = 24 m, h = 3.5 m

r = 12 m

Volume of rice = $\frac{1}{3} \pi \, 12^2 \times 3.5 = 528 \text{ m}^3$

Canvas cloth required to cover heap

$= \pi r\ell$ .... (1)

$\ell = \sqrt{12^2 + 3.5^2} = 12.50$

From (1)

Cloth required = $\frac{22}{7} \times 12 \times 12.5 = 471.43 \text{ m}^2$

27.

Salary (₹ in thousand)	Number of Persons	c.f.
5 – 10	49	49
10 – 15	133	182
15 – 20	63	245
20 – 25	15	260
25 – 30	6	266
30 – 35	7	273
35 – 40	4	277
40 – 45	2	279
45 – 50	1	280

$n = 280, \frac{n}{2} = 140$

So, median class is 10 – 15

$\ell = 10$, cf = 49, f = 133, h = 5

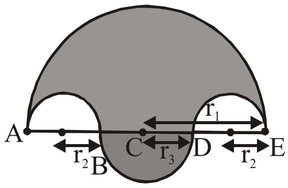
Median $= \ell + \frac{\frac{n}{2} - cf}{f} \times h$

$= 10 + \frac{140-49}{133} \times 5$

$= 10 + 3.42$

$= 13.42$

28. Let the radii of the largest semicircle, the smallest semicircle and the circle with diameter BD be r_1, r_2 and r_3 respectively.



Given, AE = 14 cm $\Rightarrow r_1 = 7$ cm

and DE = AB = 3.5 cm $\therefore r_2 = \frac{3.5}{2}$ cm

$r_3 = r_1 - 2r_2 = 7 - 2 \times \frac{3.5}{2} = 7 - 3.5 = 3.5$ cm

Area of the shaded region = Area of semicircle with radius r_1 + Area of semicircle with radius r_3 – 2 × Area of semicircle with radius r_2

$$= \frac{1}{2}\pi(r_1)^2 + \frac{1}{2}\pi(r_3)^2 - 2\times\frac{1}{2}\pi(r_2)^2$$

$$= \frac{1}{2}\pi\{(r_1)^2 + (r_3)^2 - 2(r_2)^2\}$$

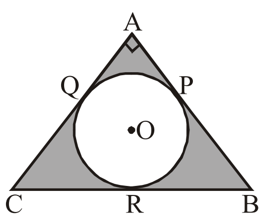
$$= \frac{1}{2}\times\frac{22}{7}\left\{(7)^2 + (3.5)^2 - 2\left(\frac{3.5}{2}\right)^2\right\}$$

$$= \frac{11}{7}\left\{49 + 12.25 - \frac{12.25}{2}\right\}$$

$$= \frac{11}{7}(49 + 6.125)$$

$$= \frac{11}{7}\times 55.125 = 86.625 \text{ cm}^2$$

OR



Given, AB = 6 cm and BC = 10 cm

By pythagoras theorem, in ΔABC, we get

$AC^2 = BC^2 - AB^2 = (10)^2 - (6)^2 = 64$

$\Rightarrow$ AC = 8 cm

Let the radius of the incircle be r.

Let the circle touch side AB at P, side AC at Q and side BC at R.

Join OP, OQ and OR.

We know that the radius from the centre of the circle is perpendicular to the tangent through the point of contact.

$\therefore$ OP $\perp$ AB, OQ $\perp$ AC and OR $\perp$ BC

Also, the tangents drawn from an external point to the circle are equal.

$\therefore$ AP = AQ, BP = BR, CR = CQ

Now, in quadrilateral

AQ = AP and $\angle AQO = \angle APO = \angle PAQ = 90°$

OPAQ is a square.

$\therefore$ OP = AQ = AP = OQ = r

$\therefore$ PB = 6 – r $\Rightarrow$ BR = 6 – r

CQ = 8 – r $\Rightarrow$ CR = 8 – r

Now, BC = BR + CR

$\Rightarrow$ 10 = 6 – r + 8 – r $\Rightarrow$ 10 = 14 – 2r

$\Rightarrow$ r = 2 cm

Now, area of shaded region

= Area of ΔABC – Area of circle

$$= \frac{1}{2}\times AB\times AC - \pi r^2 = \frac{1}{2}\times(8)\times(6) - 3.14(2)^2$$

= 24 – 12.56 = 11.44 cm²

29. Sum of all the prizes = Rs.700

Let the first prize = a

$\therefore$ 2^{nd} prize = (a – 20)

3^{rd} prize = (a – 40)

4^{th} prize = (a – 60)

Thus, we have, first term = a

Common difference = –20

Sum of 7 terms $S_7 = 700$

Since, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 700 = \frac{7}{2}[2(a) + (7-1)\times(-20)]$

$\Rightarrow 700 = \frac{7}{2}[2a + (6\times -20)]$

$\Rightarrow 700 \times \frac{2}{7} = 2a - 120$

$\Rightarrow 200 = 2a - 120 \Rightarrow 2a = 200 + 120 = 320$

$\Rightarrow a = \frac{320}{2} = 160$

Thus, the values of the seven prizes are Rs.160, Rs.(160 – 20), Rs.(160 – 40), Rs.(160 – 80), Rs.(160 – 100) and Rs.(160 – 120) = Rs.160, Rs.140, Rs.120, Rs.100, Rs.80, Rs.60 and Rs.40

30. LHS = (1 + cotA – cosecA)(1 + tanA + secA)

$= \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$

$= \left(\frac{\sin A+\cos A-1}{\sin A}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$

$= \frac{(\sin A+\cos A)^2-1}{\sin A\cos A} = \frac{1+2\sin A\cos A-1}{\sin A\cos A}$

$= \frac{2\sin A\cos A}{\sin A\cos A}$

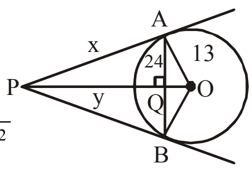
= 2

= RHS

Hence proved.

31. BQ = 12 cm,

OB = 13 cm



$\therefore$ OQ = $\sqrt{13^2 - 12^2}$

= $\sqrt{169-144}$ = $\sqrt{25}$

OQ = 5 cm

Let PQ = y and PA = x

In Δ POA : $x^2 + 13^2 = (y + 5)^2$

$x^2 + 169 = y^2 + 10y + 25$

: $x^2 - y^2 + 169 - 25 = 10y$ ... (1)

In ΔPQA : $x^2 = 12^2 + y^2$

$x^2 - y^2 = 144$ ... (2)

Put (2) in (1) 144 + 169 – 25 = 10y

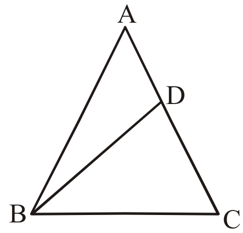
10y = 288 $\Rightarrow$ y = 28.8

PA = x = $\sqrt{144+(28.8)^2}$ = $\sqrt{973.44}$

= 31.2 cm

SECTION-D

32.



We are given that

$BC^2 = AC \times CD$

$\Rightarrow \frac{BC}{CD} = \frac{AC}{BC}$ ......(1)

In ΔABC and ΔBDC, we have

$\frac{AC}{BC} = \frac{BC}{CD}$ [Using (1)]

and $\angle$BCA = $\angle$DCB [Each = $\angle$C of ΔABC]

$\Rightarrow \Delta ABC \sim \Delta BCD$ [By SAS similarity]

$\Rightarrow \frac{AC}{BD} = \frac{BC}{CD}$ [$\because$ AB = AC is given]

$\Rightarrow \frac{AC}{BC} = \frac{BD}{CD}$

i.e., $\frac{BD}{CD} = \frac{AC}{BC}$ ....(2)

From (1) and (2), we have

$\frac{BD}{CD} = \frac{BC}{CD}$ [$\because$ Each = $\frac{AC}{BC}$]

$\Rightarrow$ BD = BC.

Hence proved

33. Let the usual speed of the train be xkm/h

$\frac{300}{x} - \frac{300}{x+5} = 2$

$\Rightarrow x^2 + 5x - 750 = 0$

$\Rightarrow (x + 30)(x - 25) = 0$

$\Rightarrow x = -30, 25$

$\therefore$ Usual speed of the train = 25 km/h

OR

$$\frac{1}{(a+b+x)} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$$

$$\Rightarrow \frac{x-a-b-x}{x(a+b+x)} = \frac{b+a}{ab}$$

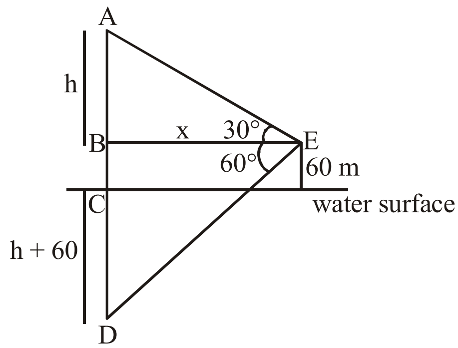
$\Rightarrow -ab = x^2 + (a + b)x$

$\Rightarrow x^2 + ax + bx + ab = 0$

$\Rightarrow (x + a)(x + b) = 0$

$\Rightarrow x = -a, -b$

34.



In ΔABE,

$$\frac{h}{x} = \tan 30°$$

$$\Rightarrow x = h\sqrt{3}$$

In ΔBDE,

$$\frac{h+60+60}{x} = \tan 60°$$

$h + 120 = x\sqrt{3}$

$h + 120 = h\sqrt{3} \times \sqrt{3}$

$2h = 120$

$h = 60$

$\therefore$ height of cloud from surface of water

$= (60 + 60)\text{m} = 120 \text{ m}$

35. Two solutions of each linear equation

$x + 3y = 6$ ...(i)

and $2x - 3y = 12$ ...(ii)

are given below.

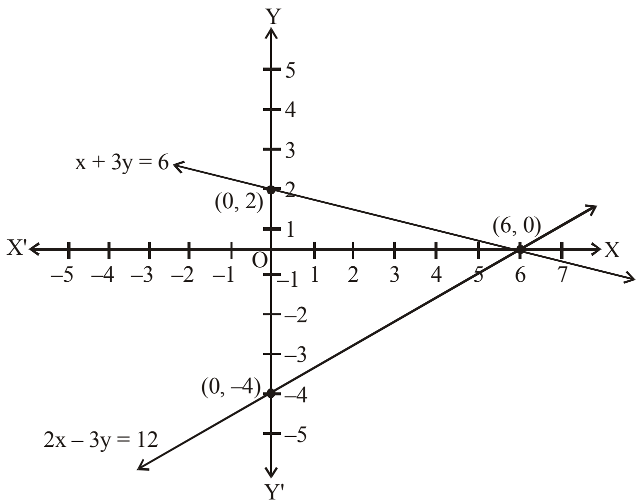
(i)

x	6	0
y	0	2

(ii)

x	6	0
y	0	– 4

The graphical representation of the given pair of linear equations is as follows :



Thus, the coordinates of point where the line $x + 3y = 6$ intersects the y-axis at (0, 2) and the line $2x - 3y = 12$ intersects the y-axis at (0, – 4).

OR

Let the fraction be $\frac{x}{y}$.

According to question

$\therefore x + y = 2x + 4 \Rightarrow x = y - 4$

Also, $\frac{x+3}{y+3} = \frac{2}{3}$

$$\Rightarrow \frac{y-4+3}{y+3} = \frac{2}{3}$$

$$\Rightarrow \frac{y-1}{y+3} = \frac{2}{3}$$

$\Rightarrow 3y - 3 = 2y + 6 \Rightarrow y = 9$

Substituting the value of y in (i), we get

$x = 5$

Thus, the required fraction is $\frac{5}{9}$.

SECTION-E

36. (i) Coordinates of $S=\left(\frac{-3+3}{2},\frac{4+4}{2}\right)$ = (0, 4)

(ii) Coordinates of $T=\left(\frac{3-2}{2},\frac{4-1}{2}\right)=\left(\frac{1}{2},\frac{3}{2}\right)$

(iii) Centriod of $\Delta PQR=\left(\frac{-3+3-2}{3},\frac{4+4-1}{3}\right)$

$=\left(\frac{-2}{3},\frac{7}{3}\right)$

Coordinates of

$U=\left(\frac{-3-2}{2},\frac{4-1}{2}\right)=\left(\frac{-5}{2},\frac{3}{2}\right)$

OR

Coordinates of Centroid of ΔSTU

$$=\left(\frac{0-\frac{5}{2}+\frac{1}{2}}{3},\frac{4+\frac{3}{2}+\frac{3}{2}}{3}\right)=\left(\frac{-2}{3},\frac{7}{3}\right)$$

37. (i) Minimum number of books = LCM(32,36)

$32 = 2 \times 2 \times 2 \times 2 \times 2$

$36 = 2 \times 2 \times 3 \times 3$

LCM $= 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$

(ii) HCF $=\frac{32\times 36}{288}=4$

OR

$36 = 2 \times 2 \times 3 \times 3$

(iii) $p = ab^2$

$= a \times b \times b$

$q = a^2b = a \times a \times b$

LCM(p, q) $= a \times a \times b \times b = a^2b^2$

38. (i) $\angle ORP = \angle OQP = 90°$

In quadrilateral ROQP

$\angle P + \angle O + \angle ORP + \angle OQP = 360°$

$\angle O = 180° - 30°$

$\angle ROQ = 150°$

(ii) $\angle RSQ = \frac{1}{2}\angle ROQ$

$=\frac{1}{2}\times 150°$

$= 75°$

(iii) In ΔORQ

OQ = OR [Radii of same circle]

$\angle OQR = \angle ORQ$ [Angle opposite to equal sides are equal]

$\angle OQR + \angle OQR + 150° = 180°$

$\angle OQR = 15°$

$\angle RQP = 90° - 15° = 75°$

OR

SR $\parallel$ PQ

$\angle SRQ = \angle RQP = 75°$ [Alternate angles]

$\angle SRO = 75° - 15° = 60°$

MATHEMATICS SAMPLE PAPER # 4

ANSWER AND SOLUTIONS

SECTION-A

1. Option (3)

0

2. Option (1)

0

3. Option (2)

2

4. Option (1)

3 : 5

5. Option (2)

$\frac{3}{4}$

6. Option (2)

14 cm

7. Option (3)

0, –2, 2

8. Option (3)

$\frac{12}{13}$

9. Option (2)

100°

10. Option (3)

$\frac{1}{2}$

11. Option (3)

60 m

12. Option (3)

60

13. Option (1)

(14, 9)

14. Option (3)

42,21

15. Option (1)

$\frac{15}{2}, 9$

16. Option (2)

–1

17. Option (1)

$$\ell + \frac{\frac{N}{2} - cf}{f} \times h$$

18. Option (1)

No real roots

19. Option (3)

Assertion (A) is true but Reason (R) is false.

20. Option (1)

Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

SECTION-B

21. Savita may have any one of the 365 days of the year as her birthday. Similarly, Hamida may have any one of 365 days of the year as her birthday.

∴ Total number of ways in which Savita and Hamida may have their birthday = 365 × 365

(i) Savita and Hamida may have same birthday on any one of 365 days of the year.

∴ Number of ways in which Savita and Hamida will have same birthday = 365

∴ Probability that Savita and Hamida will have the same birthday $= \frac{365}{365 \times 365} = \frac{1}{365}$

(ii) We have,

Probability that Savita and Hamida will have different birthdays = 1 – Probability that Savita and Hamida will have the same birthday $=1-\frac{1}{365}=\frac{364}{365}$

22. Let S_n denote the sum of n terms of an A.P. whose nth term is a_n.

We have,

$$S_n = \frac{3n^2}{2} + \frac{5n}{2}$$

$$S_{n-1} = \frac{3}{2}(n-1)^2 + \frac{5}{2}(n-1)$$

[Replacing n by (n – 1)]

$\therefore a_n = S_n - S_{n-1}$

$$= \left\{\frac{3n^2}{2}+\frac{5n}{2}\right\} - \left\{\frac{3}{2}(n-1)^2+\frac{5}{2}(n-1)\right\}$$

$$\Rightarrow a_n = \frac{3}{2}\{n^2-(n-1)^2\}+\frac{5}{2}\{n-(n-1)\}$$

$$\Rightarrow a_n = \frac{3}{2}(2n-1)+\frac{5}{2}$$

$$\Rightarrow a_{25} = \frac{3}{2}(2\times 25-1)+\frac{5}{2}=\frac{3}{2}\times 49+\frac{5}{2}=76$$

[Replacing n by 25]

OR

Here, $a_1 = -1$, $a_2 = -5$ and $d = -4$

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{16} = \frac{16}{2}[2 \times (-1) + (16-1)(-4)]$

$= 8[-2 - 60] = 8(-62)$

$= -496$

23. LHS : $\frac{\sin^4\theta+\cos^4\theta}{1-2\sin^2\theta\cos^2\theta}$

$$= \frac{(\sin^2\theta+\cos^2\theta)^2-2\sin^2\theta\cos^2\theta}{1-2\sin^2\theta\cos^2\theta}$$

$$= \frac{1-2\sin^2\theta\cos^2\theta}{1-2\sin^2\theta\cos^2\theta} = 1$$

= RHS

24. $\therefore$ ABCD is rectangle

$\Rightarrow \quad x + y = 30$ .....(1)

$x - y = 14$ ..... (2)

Adding (1) and (2) we get

$2x = 44$

$x = 22$

Subtracting (1) and (2) we get

$2y = 16$

$y = 8$

OR

The given system of equations is

$x - ky - 2 = 0$

$3x + 2y + 5 = 0$

This system of equation is of the form

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

where, $a_1 = 1$, $b_1 = -k$, $c_1 = -2$ and

$a_2 = 3$, $b_2 = 2$, $c_2 = 5$

For a unique solution, we must have

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \text{ i.e., } \frac{1}{3} \neq \frac{-k}{2} \Rightarrow k \neq \frac{-2}{3}$$

25. $\angle PAO = \angle PBO = 90°$ (angle between radius and tangent)

$\angle AOB = 105°$

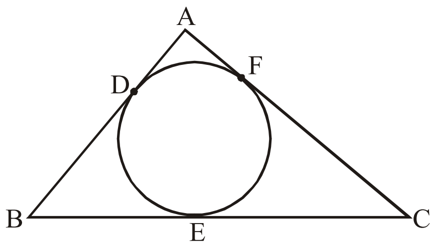
(By angle sum property of a triangle)

$$\angle AQB = \frac{1}{2} \times 105° = 52.5°$$

(Angle at the remaining part of the circle is half the angle subtended by the arc at the centre)

SECTION-C

26.



We know that tangent drawn from an external point to a circle are equal.

AF = AD

BE = BD,

CE = CF

AB = AC (Given)

AD + BD = AF + FC

$\Rightarrow$ BD = FC ($\therefore$ AD = AF)

BE = EC ($\because$ BD = BE, CE = CF)

$\therefore$ E bisects BC.

OR

AC = 8 cm

AB = 10 cm

and BC = 12 cm

Let CF = x

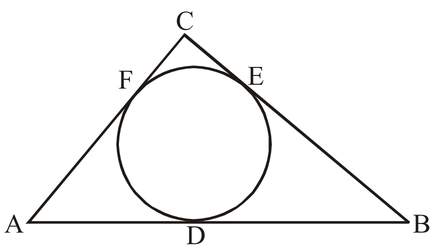
CF = EC = x

AF = 8 – x = AD

BE = 12 – x = BD

$\Rightarrow$ 8 – x + 12 – x = 10

20 – 2x = 10



–2x = –10 $\Rightarrow$ x = 5

AD = 3 cm

BE = 7 cm

and CF = 5 cm

27. Graph of 2x + 4y = 10

We have,

$2x + 4y = 10 \Rightarrow 4y = 10 - 2x \Rightarrow y = \dfrac{5-x}{2}$

When x = 1, we have

$y = \dfrac{5-1}{2} = 2$

When x = 3, we have

$y = \dfrac{5-3}{2} = 1$

Thus, we have the following table :

x	1	3
y	2	1

Graph of 3x + 6y = 12 :

We have, $3x + 6y = 12 \Rightarrow 6y = 12 - 3x$

$\Rightarrow y = \dfrac{4-x}{2}$

When x = 2, we have

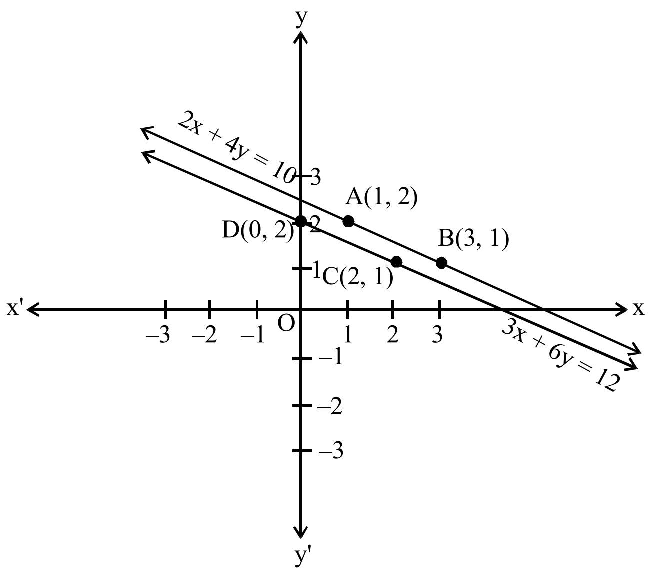
$y = \dfrac{4-2}{2} = 1$

When x = 0, we have

$y = \dfrac{4-0}{2} = 2$

Thus, we have the following table :

x	2	0
y	1	2



We find the lines represented by equations 2x + 4y = 10 and 3x + 6y = 12 are parallel. So, the two lines have no common point. Hence, the given system of equations has no solution.

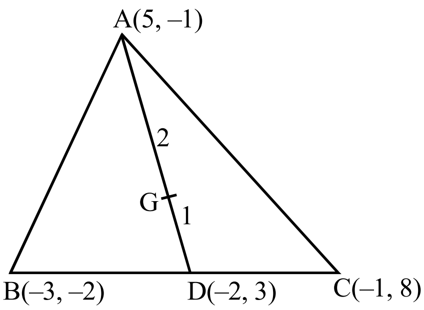
28. Let AD be the median through the vertex A of ΔABC. Then, D is the mid-point of BC. So, the coordinates of D are $\left(\frac{-3-1}{2}, \frac{-2+8}{2}\right)$

i.e.,(–2, 3).

$\therefore$ AD $= \sqrt{(5+2)^2+(-1-3)^2}$

$= \sqrt{49+16} = \sqrt{65}$ units

Let G be the centroid of ΔABC. Then, G lies on median AD and divides it in the ratio 2 : 1. So, coordinates of G are



$$\left(\frac{2\times -2+1\times 5}{2+1}, \frac{2\times 3+1\times -1}{2+1}\right)$$

$$= \left(\frac{-4+5}{3}, \frac{6-1}{3}\right) = \left(\frac{1}{3}, \frac{5}{3}\right)$$

OR

We have,

$$SP = \sqrt{(at^2-a)^2+(2at-0)^2}$$

$$= a\sqrt{(t^2-1)^2+4t^2} = a(t^2+1)$$

$$\text{and } SQ = \sqrt{\left(\frac{a}{t^2}-a\right)^2+\left(\frac{2a}{t}-0\right)^2}$$

$$\Rightarrow SQ = \sqrt{\frac{a^2(1-t^2)^2}{t^4}+\frac{4a^2}{t^2}}$$

$$\Rightarrow SQ = \frac{a}{t^2}\sqrt{(1-t^2)^2+4t^2}$$

$$= \frac{a}{t^2}\sqrt{(1+t^2)^2} = \frac{a}{t^2}(1+t^2)$$

$$\therefore \frac{1}{SP}+\frac{1}{SQ} = \frac{1}{a(t^2+1)}+\frac{t^2}{a(t^2+1)}$$

$$\Rightarrow \frac{1}{SP}+\frac{1}{SQ} = \frac{1+t^2}{a(t^2+1)} = \frac{1}{a},$$

which is independent of t.

29. LHS $= (\operatorname{cosec}\theta + \cot\theta)^2$

$= \operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta . \cot\theta.$

$$= \left(\frac{1}{\sin\theta}\right)^2 + \left(\frac{\cos\theta}{\sin\theta}\right)^2 + \frac{2\times 1}{\sin\theta}\times\frac{\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin^2\theta}+\frac{\cos^2\theta}{\sin^2\theta}+\frac{2\cos\theta}{\sin^2\theta}$$

$$= \frac{1+\cos^2\theta+2\cos\theta}{\sin^2\theta}$$

$$= \frac{(1+\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1+\cos\theta)(1+\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1+\cos\theta}{1-\cos\theta}$$

$$= \frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}}$$

$$= \frac{\sec\theta+1}{\sec\theta-1}$$

= RHS.

30. Let one number be x then other number is 27 – x

$x(27 - x) = 50$

$27x - x^2 = 50$

$x^2 - 27x + 50 = 0$

$(x - 25)(x - 2) = 0$

$x = 25$ or $x = 2$

Numbers (25, 2)

31. Let us assume, to the contrary that $\sqrt{p}$ is rational.

So, we can find co-prime integers 'a' and 'b' (b ≠ 0),

such that $\sqrt{p} = \frac{a}{b}$

$\Rightarrow \sqrt{p}\, b = a$

$\Rightarrow pb^2 = a^2$ .....(i)

$\Rightarrow a^2$ is divisible by p

$\Rightarrow$ a is divisible by p

So, we can write a = pc for some integer c.

Therefore, $a^2 = p^2c^2$ (squaring both side)

$\Rightarrow pb^2 = p^2c^2$ (from (i))

$\Rightarrow b^2 = pc^2$

$\Rightarrow b^2$ is divisible by p

$\Rightarrow$ b is divisible by p

$\Rightarrow$ p divides both a and b.

$\Rightarrow$ 'a' and 'b' have at least 'p' as a common factor

But this contradicts the fact that 'a' and 'b' are coprime.

This contradiction arises becuase we have assumed that $\sqrt{p}$ is rational.

Therefore, $\sqrt{p}$ is irrational.

SECTION-D

32. ∵ The tangents drawn to a circle from an external point are equal.

∴ AP = AC, Join OC

In ΔPAO and ΔCAO, we have:

AO = AO [Common]

OP = OC [Radii of the same circle]

AP = AC [Proved above]

$\Rightarrow \Delta PAO \cong \Delta AOC$ [SSS congruency]

$\therefore \angle PAO = \angle CAO$

$\Rightarrow \angle PAC = 2\angle CAO$ ...(1)

Similarly $\angle CBQ = 2\angle CBO$ ...(2)

Again, we know that sum of internal anngles on the same side of a transversal is 180°.

$\therefore \angle PAC + \angle CBQ = 180°$

$\Rightarrow 2\angle CAO + 2\angle CBO = 180°$

[From (1) and (2)]

$\Rightarrow \angle CAO + \angle CBO = \frac{180°}{2} = 90°$ ...(3)

Also in ΔAOB, $\angle BAO + \angle ABO + \angle AOB = 180°$

[Sum of angles of a triangle]

$\Rightarrow \angle CAO + \angle CBO + \angle AOB = 180°$ [By (3)]

$\Rightarrow 90° + \angle AOB = 180°$

$\Rightarrow \angle AOB = 180° - 90°$

$\Rightarrow \angle AOB = 90°$.

33. Let P and Q be the two positions of the plane and let A be point of observation. Let ABC be the horizontal line through A. It is given that angles of elevation of the plane in two positions P and Q from a point A are 60° and 30° respectively.

$\therefore \angle PAB = 60°, \angle QAB = 30°$

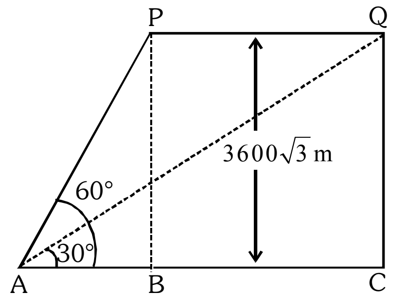
It is also given that $PB = 3600\sqrt{3}$ m

In ΔABP, we have

$\tan 60° = \dfrac{PB}{AB}$

$\sqrt{3} = \dfrac{3600\sqrt{3}}{AB} \Rightarrow AB = 3600$ m

In ΔACQ,



$\tan 30° = \dfrac{QC}{AC}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{3600\sqrt{3}}{AC}$

$\Rightarrow AC = 10800$ m

$\therefore PQ = BC = AC - AB = 10800\text{ m} - 3600\text{ m}$

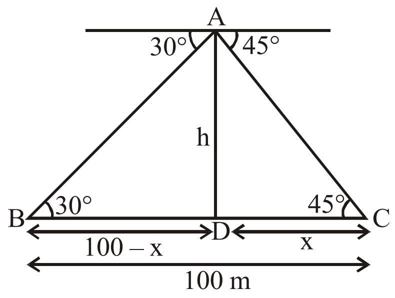
$= 7200$ m

Thus the plane travels 7200 m in for 30 seconds.

Hence, speed of plane $= \dfrac{7200}{30} = 240$ m/s

$= \dfrac{240}{1000} \times 60 \times 60 = 864$ km/hr

OR



Let AD be the height (h) of the light house and BC is the distance between the ships and DC = x(let)

Given, BC = 100 m

$\tan 45° = \dfrac{h}{x}$

$\Rightarrow x = h$

In ΔABD, $\tan 30° = \dfrac{h}{100 - DC}$

$\Rightarrow \dfrac{1}{\sqrt{3}} = \dfrac{h}{100 - x}$

$\therefore 100 - x = h\sqrt{3}$

$100 - h = h\sqrt{3}$

$100 - h = h\sqrt{3}$

$\Rightarrow 100 = h + h\sqrt{3}$ [By (i)]

$\Rightarrow 100 = h(1+\sqrt{3})$

$h = \dfrac{100}{1+\sqrt{3}}$

$\Rightarrow h = \dfrac{100(\sqrt{3}-1)}{3-1}$

$= 50(\sqrt{3} - 1)$

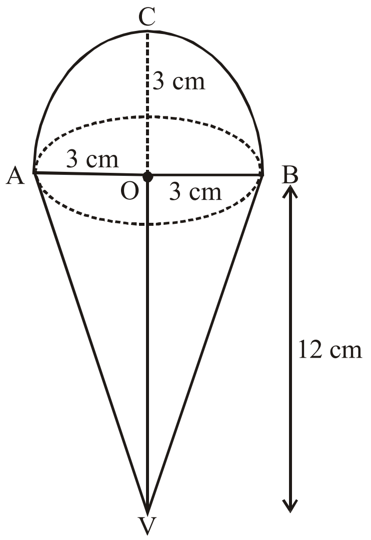
$= 50(1.732 - 1)$

$= 50 \times 0.732$

$\therefore$ Height of tower = 36.6 m

34. We have,

Volume of ice-cream in the container shaped like a right circular cylinder having radius 6 cm and height 15 cm = $\pi \times 6^2 \times 15$ cm^3



Volume of one ice-cream cone shown in figure

$$= \left\{\frac{2}{3}\pi \times 3^3 + \frac{1}{3}\pi \times 3^2 \times 12\right\} \text{cm}^2$$

$= (18\pi + 36\pi)$ cm^3 = 54π cm^3

Let the total number of cones that can be filled with the ice-cream given in the container be n. Then,

Volume of ice-cream in n cones = Volume of ice-cream in the container

$\Rightarrow 54\pi \times n = \pi \times 36 \times 15$

$$\Rightarrow n = \frac{\pi \times 36 \times 15}{54\pi} = 10$$

35. The frequency distribution table of the given data can be drawn as :

Class	x_i	f_i	$f_i x_i$	cf
0 – 50	25	2	50	2
50 – 100	75	3	225	5
100 – 150	125	5	625	10
150 – 200	175	6	1050	16
200 – 250	225	5	1125	21
250 – 300	275	3	825	24
300 – 350	325	1	325	25
		$\Sigma f_i = 25$	$\Sigma f_i x_i = 4225$	

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{4225}{25} = 169$$

$$\text{Median} = \ell + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

Since, $\dfrac{\Sigma f_i}{2} = \dfrac{n}{2} = \dfrac{25}{2}$ = 12.5. This observation lies in the class 150 – 200.

$\therefore$ Lower limit of median class (ℓ) = 150

Class size (h) = 50

Cumulative frequency (cf) = 10

Frequency of median class (f) = 6

$$\therefore \text{Median} = 150 + \left[\frac{\frac{25}{2} - 10}{6}\right] \times 50 = 150 + \left[\frac{2.5}{6}\right] \times 50$$

= 150 + 20.83 = 170.83

Mode = 3 Median – 2 Mean

= 3 (170.83) – 2(169) = 512.49 – 338 = 174.49

OR

C.I.	f_i	c.f.	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
05 – 07	70	70	6	–3	–210
07 – 09	120	190	8	–2	–240
09 – 11	32	222	10	–1	–32
11 – 13	100	322	12	0	0
13 – 15	45	367	14	1	45
15 – 17	28	395	16	2	56
17 – 19	5	400	18	3	15
	$\sum f = 400$				$\sum f_i u_i = -366$

a = Assumed mean = 12

Mean, $\bar{x} = a + \frac{\sum f_i u_i}{\sum f_i} \times h$

Mean = $12 + \frac{-366}{400} \times 2 = 10.17$

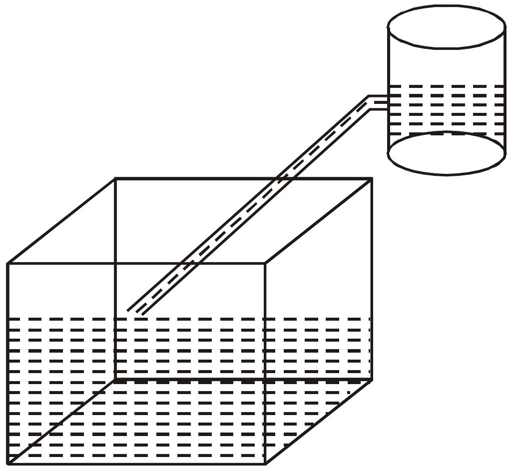
$\frac{\sum f}{2} = 200 \Rightarrow$ Median class = 09 – 11

Median = $\ell + \left(\frac{\frac{n}{2} - \text{c.f.}}{f}\right) \times h$

$\Rightarrow$ Median = $9 + \frac{200 - 190}{32} \times 2 = 9.625$

SECTION-E

36.



(i) $\frac{\text{Capacity of sump}}{\text{Capacity of tank}} = \frac{\ell bh}{\pi r^2 H}$

$= \frac{1.57 \times 1.44 \times 0.95}{3.14 \times 0.6 \times 0.6 \times 0.95} = \frac{2}{1}$

(ii) C.S.A. of cylindrical tank = $2\pi rH$

= 2 × 3.14 × 60 × 95

= 35796 cm^2

= 3.5796 m^2

= 3.6 m^2

(iii) Volume of water in cylindrical tank

= $\pi r^2 h$

= 3.14 × 60 × 60 × 95

= 1073880 cm^3

Now, 1ℓ = 1000 cm^3

∴ Volume of tank = 1073.88 ℓ

20ℓ tank is filled in 1 minute

∴ 1073.88 ℓ tank is filled in $\frac{1073.88\ell}{20}$

= 53.69

= 54 minutes

OR

Volume of water in sump = 1500 litres

= 1500 litres

= 1.5 m^3

Then, $V = \ell bh$

$1.57 \times 1.44 \times h = 1.5$

$h = \frac{1.5}{1.57 \times 1.44} = 0.663$ m

= 66.3 cm

37. (i) Suppose two cars meet at point Q.

Then, Distance travelled by car X = AQ.

Distance travelled by car Y = BQ.

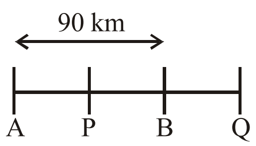
It is given that two cars meet in 9 hours

Distance travelled by car X in 9 hours

= 9x km = AQ = 9x

Distance travelled by car Y in 9 hours

= 9y km = BQ = 9y



Clearly, AQ – BQ = AB

$\Rightarrow 9x - 9y = 90$

$\Rightarrow x - y = 10$

OR

Suppose two cars meet at point P.

Then Distance travelled by car X = AP

and Distance travelled by car Y = BP.

In this case, two cars meet in $\frac{9}{7}$ hours

$= \frac{9}{7}$ xkm

$\Rightarrow AP = \frac{9}{7}x$

Distance travelled by car Y in $\frac{9}{7}$ hours

$= \frac{9}{7}y$ km

Clearly, AP + BP = AB

$\Rightarrow \frac{9}{7}x + \frac{9}{7}y = 90$

$\Rightarrow \frac{9}{7}(x + y) = 90$

$\Rightarrow x + y = 70$

(ii) We have $x - y = 10$

$\Rightarrow x + y = 70$

Adding equations (i) and (ii), we get

$2x = 80$

$\Rightarrow x = 40$

Hence, speed of car X is 40 km/hr.

(iii) We have $x - y = 10$

$\Rightarrow 40 - y = 10$

$\Rightarrow y = 30$

Hence, speed of car Y is 30 km/hr

38. (i) Scale factor $= \frac{AC}{AE}$

$= \frac{AC}{AC + CE} = \frac{8}{8+4}$

$= \frac{8}{12} = \frac{2}{3}$

(ii) Since, $\Delta EBC \sim \Delta EFA$

$\frac{EC}{EA} = \frac{BC}{AF}$

$\Rightarrow \frac{4}{12} = \frac{3.6}{AF}$

$\Rightarrow AF = 3.6 \times 3$

= 10.8 cm

(iii) $\Delta ABC \sim \Delta ADE$

$$\frac{AC}{AE} = \frac{BC}{DE}$$

$$\frac{8}{12} = \frac{3.6}{DE}$$

$$DE = \frac{3.6 \times 3}{2} = 5.4 \text{ cm}$$

OR

$$\frac{AB}{BD} = \frac{AC}{CE}$$

$$\frac{AB}{BD} = \frac{8}{4}$$

$$\frac{AB}{BD} = \frac{2}{1}$$