JEE(ADVANCED)–2024 (EXAMINATION)

(Held On Sunday 26th MAY, 2024)

PHYSICS | **TEST PAPER WITH ANSWER AND SOLUTION**

PAPER-1

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −1 In all other cases.

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1. A dimensionless quantity is constructed in terms of electronic charge e, permittivity of free space ε_0, Planck's constant h, and speed of light c. If the dimensionless quantity is written as $e^{\alpha}\varepsilon_0^{\beta}h^{\gamma}c^{\delta}$ and n is a non-zero integer, then $(\alpha, \beta, \gamma, \delta)$is given by

(A) $(2n, -n, -n, -n)$

(B) $(n, -n, -2n, -n)$

(C) $(n, -n, -n, -2n)$

(D) $(2n, -n, -2n, -2n)$

Ans. **(A)**

Sol. For the quantity to be dimensionless

$$e^{\alpha}\varepsilon_0^{\beta}h^{\gamma}c^{d} = M^0L^0T^0A^0$$

$$\Rightarrow (AT)^{\alpha}\left(M^{-1}L^{-3}T^{4}A^{2}\right)^{\beta}\left(ML^{2}T^{-1}\right)^{\gamma}\left(LT^{-1}\right)^{\delta} = A^0M^0L^0T^0$$

$$\therefore\ \alpha + 2\beta = 0,\ \alpha + 4\beta - \gamma - \delta = 0,\ -\beta + \gamma = 0\ \&\ -3\beta + 2\gamma + \delta = 0$$

$$\therefore\ \alpha = -2\beta,\ \beta = \gamma\ \&\ \gamma = \delta$$

$\therefore$ Option (A) satisfies the given condition

2. An infinitely long wire, located on the z-axis, carries a current I along the +z-direction and produces the magnetic field $\vec{B}$. The magnitude of the line integral $\int \vec{B}.\vec{dl}$ along a straight line from the point $(-\sqrt{3}a, a, 0)$ to $(a, a, 0)$ is given by

[μ_0 is the magnetic permeability of free space.]

(A) $7\mu_0 I/24$

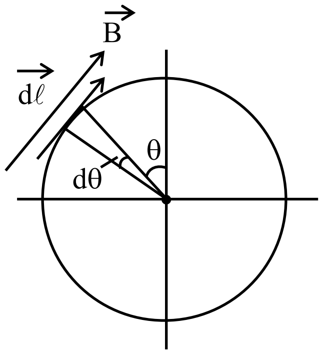
(B) $7\mu_0 I/12$

(C) $\mu_0 I/8$

(D) $\mu_0 I/6$

Ans. **(A)**

Sol.



$\Rightarrow |\vec{d\ell}| = rd\theta$

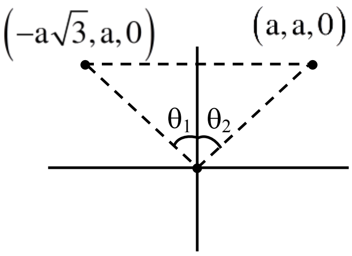
$\Rightarrow |\vec{B}| = \dfrac{\mu_0 I}{2\pi r}$

$\Rightarrow \int \vec{B}\cdot\vec{d\ell} = \int |\vec{B}||\vec{d\ell}|\cos 0°$

$= \int\left(\dfrac{\mu_0 I}{2\pi r}\right)\times(rd\theta)$

$= \int_{\theta_1}^{\theta_2}\dfrac{\mu_0 I}{2\pi}d\theta = \dfrac{\mu_0 I}{2\pi}\left[\theta_2 - (-\theta_1)\right]$

[θ_1 is anticlockwise hence taken negative]



$\Rightarrow \tan\theta_1 = \dfrac{a\sqrt{3}}{a} = \sqrt{3}$

$\theta_1 = \dfrac{\pi}{3}$

$\Rightarrow \tan\theta_2 = \dfrac{a}{a} = 1$

$\theta_2 = \dfrac{\pi}{4}$

$\Rightarrow \int B\cdot d\ell = \dfrac{\mu_0 I}{2\pi}\left[\dfrac{\pi}{3}+\dfrac{\pi}{4}\right]$

$= \dfrac{7\mu_0 I}{24}$

$\Rightarrow$ Ans. Option (A)

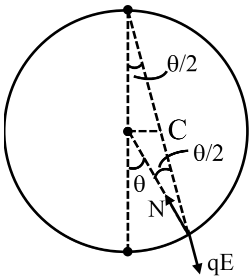
3. Two beads, each with charge q and mass m, are on a horizontal, frictionless, non-conducting, circular hoop of radius R. One of the beads is glued to the hoop at some point, while the other one performs small oscillations about its equilibrium position along the hoop. The square of the angular frequency of the small oscillations is given by [ε_0 is the permittivity of free space]

(A) $q^2 / (4\pi\varepsilon_0 R^3 m)$ (B) $q^2 / (32\pi\varepsilon_0 R^3 m)$

(C) $q^2 / (8\pi\varepsilon_0 R^3 m)$ (D) $q^2 / (16\pi\varepsilon_0 R^3 m)$

Ans. **(B)**

Sol.



Restoring force = $qE\sin\left(\frac{\theta}{2}\right)$

$\therefore \tau = qE\sin\left(\frac{\theta}{2}\right)R = I\alpha$

$$E = \frac{Kq}{\left(2R\cos\frac{\theta}{2}\right)^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{4R^2\cos^2\left(\frac{\theta}{2}\right)}$$

$$\therefore \frac{1}{4\pi\epsilon_0}\frac{qR}{4R^2\cos^2\left(\frac{\theta}{2}\right)}\sin\left(\frac{\theta}{2}\right)q = mR^2\alpha$$

For θ very small,

$$\frac{-q^2}{32\pi\varepsilon_0 R^3 m}\theta = \alpha$$

$$\therefore \omega^2 = \frac{q^2}{32\pi\varepsilon_0 mR^3}$$

Hence option (B)

4. A block of mass 5 kg moves along the x-direction subject to the force $F = (-20x + 10)$ N, with the value of x in metre. At time $t = 0$ s, it is at rest at position $x = 1$ m. The position and momentum of the block at $t = (\pi/4)$ s are

(A) –0.5 m, 5 kg m/s (B) 0.5 m, 0 kg m/s (C) 0.5 m, –5 kg m/s (D) –1 m, 5 kg m/s

Ans. **(C)**

Sol. $F = -20\left(x - \frac{1}{2}\right) = -20X \qquad \left(X = x - \frac{1}{2}\right)$

$\therefore$ Particle will perform SHM about $x = \frac{1}{2}$ with

$\omega = 2$ rad/sec $\Rightarrow T = \pi$ sec.

$\therefore$ Phase covered in $t = \frac{\pi}{4}$ second $= 90°$.

Given particle is at rest at x = 1m $\Rightarrow$ x = 1 is extreme position.

$\therefore$ In $\frac{\pi}{4}$ sec, it will be at equilibrium

$\therefore$ x = 0.5 m and momentum = $m\omega A = 5 \times 2 \times 0.5 = 5$ kg m/s

Direction will be towards –ve x.

Hence option (C)

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks	: +4	**ONLY** if (all) the correct option(s) is(are) chosen;
Partial Marks	: +3	If all the four options are correct but **ONLY** three options are chosen;
Partial Marks	: +2	If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
Partial Marks	: +1	If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
Zero Marks	: 0	If none of the options is chosen (i.e. the question is unanswered);
Negative Marks	: −2	In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then

  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

5. A particle of mass m is moving in a circular orbit under the influence of the central force $F(r) = -kr$, corresponding to the potential energy $V(r) = kr^2 / 2$, where k is a positive force constant and r is the radial distance from the origin. According to the Bohr's quantization rule, the angular momentum of the particle is given by $L = n\hbar$, where $\hbar = h/(2\pi)$, h is the Planck's constant, and n a positive integer. If v and E are the speed and total energy of the particle, respectively, then which of the following expression(s) is(are) correct?

(A) $r^2 = n\hbar\sqrt{\frac{1}{mk}}$ (B) $v^2 = n\hbar\sqrt{\frac{k}{m^3}}$

(C) $\frac{L}{mr^2} = \sqrt{\frac{k}{m}}$ (D) $E = \frac{n\hbar}{2}\sqrt{\frac{k}{m}}$

Ans. **(A,B,C)**

Sol. The central force will provide necessary centripetal force

$$\Rightarrow kr = \frac{mv^2}{r}$$

or, $kr^2 = mv^2$ …. (1)

By quantisation rule

$n\hbar = mvr$

or, $\frac{n\hbar}{r} = mv$ … (2)

$$\frac{(1)}{(2)^2} \Rightarrow \frac{kr^2}{\frac{n^2\hbar^2}{r^2}} = \frac{mv^2}{m^2v^2}$$

$$\Rightarrow \frac{k}{n^2\hbar^2}r^4 = \frac{1}{m}$$

$$\Rightarrow r = \left(\frac{n^2\hbar^2}{km}\right)^{\frac{1}{4}} \Rightarrow r^2 = \frac{n\hbar}{\sqrt{mk}}$$

(B) Using (1), $K \cdot \frac{n\hbar}{\sqrt{mk}} = mv^2$

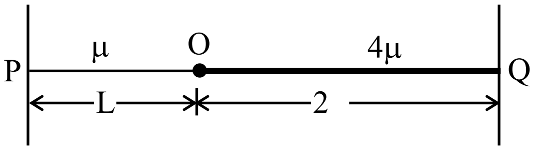
$$\Rightarrow v^2 = n\hbar\sqrt{\frac{k}{m^3}}$$

(C) $\frac{L}{mr^2} = \frac{mvr}{mr^2} = \frac{v}{r} = \sqrt{\frac{k}{m}}$ from (1)

(D) $E = \frac{1}{2}mv^2 + \frac{1}{2}kr^2 = \frac{n\hbar}{2}\sqrt{\frac{k}{m}} + \frac{1}{2}k\frac{n\hbar}{\sqrt{mk}}$

$$E = n\hbar\sqrt{\frac{k}{m}}$$

6. Two uniform string of mass per unit length μ and 4μ, and length L and $2L$, respectively, are joined at point O, and tied at two fixed ends P and Q, as shown in the figure. The strings are under a uniform tension T. If we define the frequency $v_0 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$, which of the following statement(s) is (are) correct ?



(A) With a node at O, the minimum frequency of vibration of the composite string is v_0.

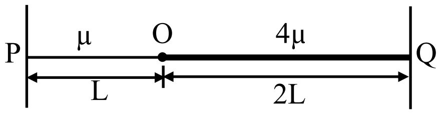
(B) With an antinode at O, the minimum frequency of vibration of the composite string is $2v_0$.

(C) When the composite string vibrates at the minimum frequency with a node at O, it has 6 nodes, including the end nodes.

(D) No vibrational mode with an antinode at O is possible for the composite string.

Ans. **(A,C,D)**

Sol.



$$C_1 = \sqrt{\frac{T}{\mu}}, C_2 = \sqrt{\frac{T}{4\mu}} = \frac{C_1}{2}$$

<u>For node at O :</u>

$$L = \frac{n\lambda_1}{2}, 2L = \frac{m\lambda_2}{2} \text{ (n, m are integers)}$$

$$\lambda_1 = \frac{2L}{n}, \lambda_2 = \frac{4L}{m}$$

$$\frac{C_1}{\lambda_1} = \frac{C_2}{\lambda_2}$$

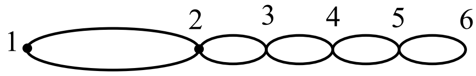
$$\Rightarrow \frac{C_1}{\frac{2L}{n}} = \frac{\frac{C_1}{2}}{\frac{4L}{m}}$$

$$\Rightarrow 4n = m$$

For minimum frequency, n = 1, m = 4

$$\therefore \nu_{min} = \frac{C_1 \times 1}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \nu_0$$

The string will look like



Total no. of nodes = 6 including the end nodes

For antinode at O :

$$L=(2n+1)\frac{\lambda_1}{4};\ 2L=(2n+1)\frac{\lambda_2}{4} \qquad \text{(n, m are integers)}$$

$$\lambda_1=\frac{4L}{(2n+1)};\ \lambda_2=\frac{8L}{(2m+1)}$$

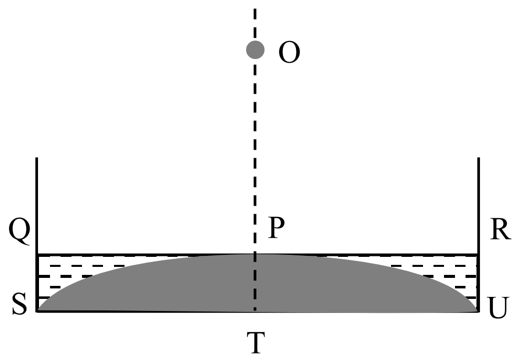
$$\frac{C_1}{\lambda_1}=\frac{C_2}{\lambda_2}$$

$$\frac{C_1}{C_2}=\frac{\lambda_1}{\lambda_2}$$

$$2=\frac{\frac{4L}{(2n+1)}}{\frac{8L}{(2m+1)}}$$

$$4=\frac{(2m+1)}{(2n+1)} \Rightarrow \text{even}=\frac{\text{odd}}{\text{odd}} \Rightarrow \text{This node is not possible}$$

7. A glass beaker has a solid, plano-convex base of refractive index 1.60, as shown in the figure. The radius of curvature of the convex surface (SPU) is 9 cm, while the planar surface (STU) acts as a mirror. This beaker is filled with a liquid of refractive index n up to the level QPR. If the image of a point object O at a height of h (OT in the figure) is formed onto itself, then, which of the following option(s) is(are) correct?



(A) For n = 1.42, h = 50 cm.

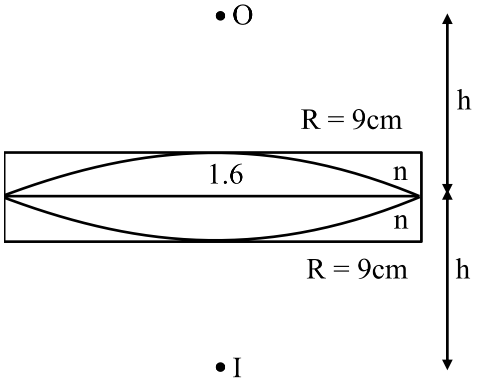
(B) For n = 1.35, h = 36 cm.

(C) For n = 1.45, h = 65 cm.

(D) For n = 1.48, h = 85 cm.

Ans. **(A,B)**

Sol. Since STU is a plane mirror, we can take mirror image of the whole situation about it and final image can be assumed to be at a distance h below the base.



Since object and image are at same distance from equivalent lens, hence h = $2F_{eq}$

$$\frac{1}{F_{eq}} = \left(\frac{1.6-1}{1}\right)\left(\frac{2}{9}\right) + \frac{(n-1)}{1}\left(\frac{-2}{9}\right)$$

$$\frac{1}{\frac{h}{2}} = \frac{1.2}{9} + \frac{2(1-n)}{9}$$

$$\frac{2}{h} = \frac{3.2-2n}{9}$$

$$h = \frac{9}{1.6-n}\text{cm}$$

(A) for n = 1.42, h = 50 cm

(B) for n = 1.35, h = 36 cm

(C) for n = 1.45, h = 60 cm

(D) for n = 1.48, h = 75 cm

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** If the correct integer is entered;

Zero Marks : 0 In all other cases.

8. The specific heat capacity of a substance is temperature dependent and is given by the formula $C = kT$, where k is a constant of suitable dimensions in SI units, and T is the absolute temperature. If the heat required to raise the temperature of 1 kg of the substance from – 73°C to 27°C is nk, the value of n is ______.

[Given : 0 K = –273 °C.]

Ans. **(25000)**

Sol. T_i = – 73°C = 200 K

T_f = 27°C = 300 K

$Q = \int msdT$

$= \int 1 \cdot kT\ dT$

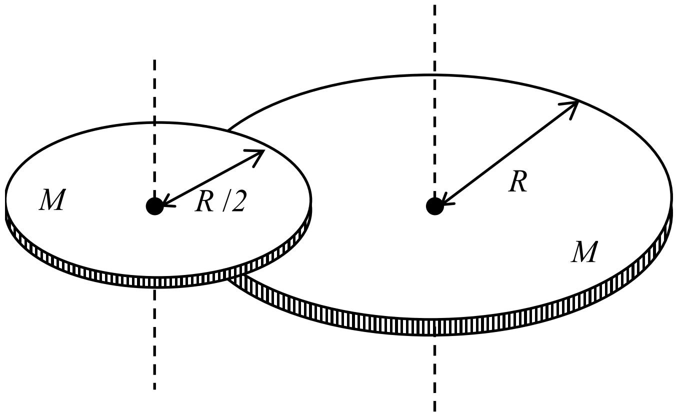
$= \int kT\ dT\ = K\int_{200}^{300} T\ dT$

$= \frac{K}{2}\left[T^2\right]_{200}^{300} = \frac{K}{2}\left[300^2 - 200^2\right]$

Q = 25000 K

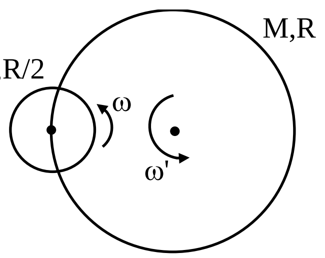
Hence η = 25000

9. A disc of mass *M* and radius *R* is free to rotate about its vertical axis as shown in the figure. A battery operated motor of negligible mass is fixed to this disc at a point on its circumference. Another disc of the same mass *M* and radius *R* / 2 is fixed to the motor's thin shaft. Initially, both the discs are at rest. The motor is switched on so that the smaller disc rotates at a uniform angular speed ω. If the angular speed at which the large disc rotates is ω / *n*, then the value of *n* is ______.



Ans. **(12)**

Sol.



On applying conservation of angular momentum about axis of larger disc.

$$\frac{1}{2}\cdot M\left(\frac{R}{2}\right)^2\cdot\omega - M(\omega' R)\cdot R - \frac{MR^2}{2}\cdot\omega' = 0$$

$$\Rightarrow \frac{\omega}{8} = \frac{3\omega'}{2}$$

$$\Rightarrow \omega' = \frac{\omega}{12}$$ Hence, n = 12

10. A point source S emits unpolarized light uniformly in all directions. At two points A and B, the ratio $r = I_A / I_B$ of the intensities of light is 2. If a set of two polaroids having 45° angle between their pass-axes is placed just before point B, then the new value of r will be _____.

Ans. **(8)**

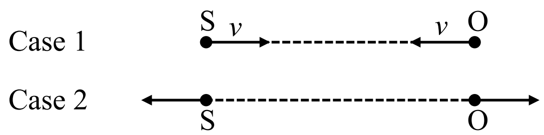
Sol. New intensity at B

$$I'_B = \left(\frac{I_B}{2}\right)\cos^2 45^\circ = \frac{I_B}{4}$$

New value of $\alpha = \frac{I_A}{\frac{I_B}{4}} = \frac{4I_A}{I_B}$

$= 4 \times 2; \alpha = 8$

11. A source (S) of sound has frequency 240 Hz. When the observer (O) and the source move towards each other at a speed v with respect to the ground (as shown in Case 1 in the figure), the observer measures the frequency of the sound to be 288 Hz. However, when the observer and the source move away from each other at the same speed v with respect to the ground (as shown in Case 2 in the figure), the observer measures the frequency of sound to be n Hz. The value of n is _____.



Ans. **(200)**

Sol. Frequency received by observer

$$f_0 = \left(\frac{C \pm V_0}{C \pm V_S}\right) f_S$$, C is speed of sound

Case-1:

$$f_1 = \left(\frac{C + V}{C - V}\right) f_S$$

$$288 = \left(\frac{C + V}{C - V}\right) 240$$

Case-2:

$$f_2 = \left(\frac{C - V}{C + V}\right) f_S$$

$$n = \left(\frac{C - V}{C + V}\right) 240$$

multiply the two equations, we get.

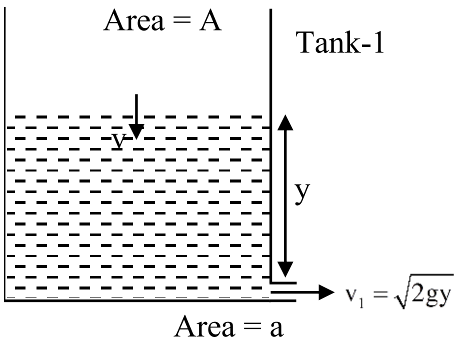
(288) (n) = (240) (240)

N = 200

12. Two large, identical water tanks, 1 and 2, kept on the top of a building of height H, are filled with water up to height h in each tank. Both the tanks contain an identical hole of small radius on their sides, close to their bottom. A pipe of the same internal radius as that of the hole is connected to tank 2, and the pipe ends at the ground level. When the water flows from the tanks 1 and 2 through the holes, the times taken to empty the tanks are t_1 and t_2, respectively. If $H = \left(\frac{16}{9}\right)h$, then the ratio t_1/t_2 is _____.

Ans. **(3)**

Sol.

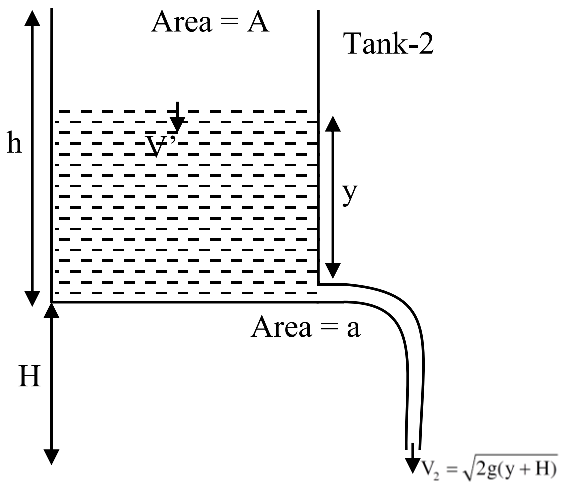


$Av = av_1$

$$A\left(-\frac{dy}{dt}\right) = a\sqrt{2gy}\ ;\ dt = \frac{A}{a\sqrt{2g}}.\frac{-dy}{\sqrt{y}}$$

$$\int_0^{t_1} dt = \frac{A}{a\sqrt{2g}}\int_h^0 -\frac{dy}{\sqrt{y}}$$

$$t_1 = \frac{A}{a\sqrt{2g}}2\sqrt{h}\ ;\ t_1 = \frac{A}{a}\sqrt{\frac{2h}{g}}$$



$Av' = av_2$

$$A\left(-\frac{dy}{dt}\right) = a\sqrt{2g(H+y)}$$

$$dt = -\frac{A}{a\sqrt{2g}}\frac{dy}{\sqrt{H+y}}$$

$$\int_0^{t_2} dt = -\frac{A}{a\sqrt{2g}}\int_H^0 \frac{dy}{\sqrt{H+y}}$$

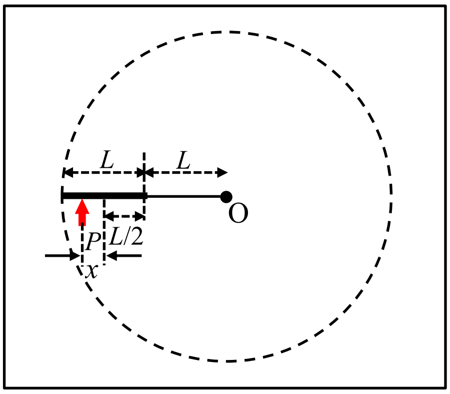
$$t_2 = \frac{A}{a\sqrt{2g}}(2)(\sqrt{H+h} - \sqrt{H}) \ \& \ H = \frac{16h}{9}$$

$$= \frac{A}{a}\sqrt{\frac{2h}{g}}\left(\frac{5}{3}-\frac{4}{3}\right)$$

$$t_2 = \frac{A}{a}\sqrt{\frac{2h}{g}}\left(\frac{1}{3}\right)$$

ratio $\frac{t_1}{t_2} = 3$

13. A thin uniform rod of length L and certain mass is kept on a frictionless horizontal table with a massless string of length L fixed to one end (top view is shown in the figure). The other end of the string is pivoted to a point O. If a horizontal impulse P is imparted to the rod at a distance $x = L/n$ from the mid-point of the rod (see figure), then the rod and string revolve together around the point O, with the rod remaining aligned with the string. In such a case, the value of n is ______.



Ans. **(18)**

Sol. Linear impulse $\int F dt = \Delta$ momentum

$= m\,(V_{cm} - 0)$

$P = m\,(\omega\, r_{cm})$

$$= m\omega\left(L + \frac{L}{2}\right)$$

$$P = m\omega\left(\frac{3L}{2}\right) \qquad \ldots(i)$$

Angular impulse $\int \tau dt = \Delta$ angular momentum

$\int r \times F dt = \Delta L$

$r \times \int F dt = I(\omega - 0)$, and I is moment of inertia about axis of rotation.

$$\left(L + \frac{L}{2} + x\right) \times P = (I_{cm} + md^2)\omega$$

$$= \left(\frac{mL^2}{12} + m\left(L + \frac{L}{2}\right)^2\right)\omega$$

$$\left(\frac{3L}{2} + x\right)P = mL^2\left(\frac{1}{12} + \left(\frac{3}{2}\right)^2\right)\omega$$

$$\left(\frac{3L}{2} + x\right)P = mL^2\left(\frac{7}{3}\right)\omega \qquad \text{....(ii)}$$

Divide eq.-(i) & (ii)

$$\left(\frac{3L}{2} + x\right) = \frac{L\left(\frac{7}{3}\right)}{\left(\frac{3}{2}\right)}$$

$$\frac{3L}{2} + x = L\left(\frac{14}{9}\right)$$

$$x = \frac{L}{18}$$

SECTION-4 : (Maximum Marks : 12)

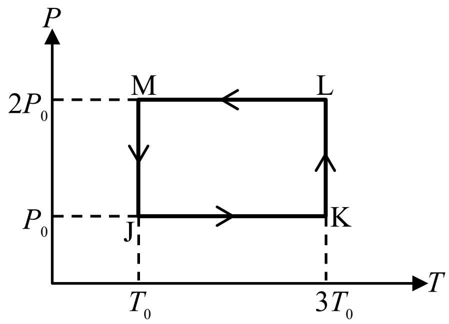
- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : –1 In all other cases.

14. One mole of a monatomic ideal gas undergoes the cyclic process $J \to K \to L \to M \to J$, as shown in the P-T diagram.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option. [R is the gas constant.]

	List-I	List-II
(P)	Work done in the complete cyclic process	(1) $RT_0 - 4RT_0 \ln 2$
(Q)	Change in the internal energy of the gas in the process JK	(2) 0
(R)	Heat given to the gas in the process KL	(3) $3RT_0$
(S)	Change in the internal energy of the gas in the process MJ	(4) $-2RT_0 \ln 2$
		(5) $-3RT_0 \ln 2$

(A) $P \to 1; Q \to 3; R \to 5; S \to 4$ (B) $P \to 4; Q \to 3; R \to 5; S \to 2$

(C) $P \to 4; Q \to 1; R \to 2; S \to 2$ (D) $P \to 2; Q \to 5; R \to 3; S \to 4$

Ans. **(B)**

Sol. $J(P_0, V_0, T_0)$

$K(P_0, 3V_0, 3T_0)$

$M\left(2P_0, \frac{V_0}{2}, T_0\right)$

$L\left(2P_0, \frac{3V_0}{2}, 3T_0\right)$

$P_0V_0 = nRT_0$

$JK \to$ isobaric $\Rightarrow W = P_0(2V_0) = 2nRT_0$

$$\Delta U = \frac{3}{2}nR(2T_0) = 3nRT_0$$

$$KL \to \text{isothermal} \to W = nR(3T)\ \ell n\left(\frac{1}{2}\right) = -3nRT_0\ell n2$$

$\Delta U = 0 \Rightarrow Q = -3nRT_0\ell n2$

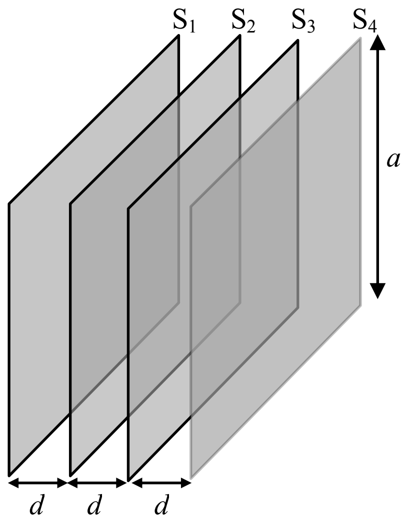
$LM \to$ isobaric $= 2P_0(-V_0) = -2nRT_0$

$MJ \to$ isothermal $\Rightarrow nRT_0\ell n2;\ \Delta U = 0$

$WD_{net} = -2nRT_0\ \ell n2$

$P \to 4, Q \to 3, R \to 5, S \to 2$

15. Four identical thin, square metal sheets, S_1, S_2, S_3 and S_4, each of side a are kept parallel to each other with equal distance d ($<< a$) between them, as shown in the figure. Let $C_0 = \varepsilon_0 a^2/d$, where ε_0 is the permittivity of free space.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

	List-I	List-II
(P)	The capacitance between S_1 and S_4, with S_2 and S_3 not connected, is	(1) $3C_0$
(Q)	The capacitance between S_1 and S_4, with S_2 shorted to S_3, is	(2) $C_0/2$
(R)	The capacitance between S_1 and S_3, with S_2 shorted to S_4, is	(3) $C_0/3$
(S)	The capacitance between S_1 and S_2, with S_3 shorted to S_1, and S_2 shorted to S_4, is	(4) $2C_0/3$
		(5) $2C_0$

(A) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 4; S $\rightarrow$ 5

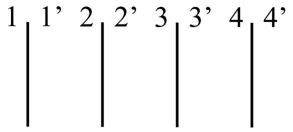
(B) P $\rightarrow$ 2; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 1

(C) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 4; S $\rightarrow$ 1

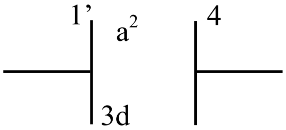
(D) P $\rightarrow$ 3; Q $\rightarrow$ 2; R $\rightarrow$ 2; S $\rightarrow$ 5

Ans. (C)

Sol.

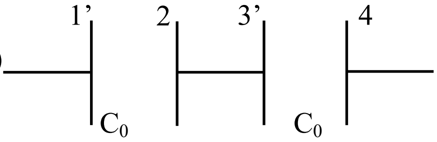


(P)



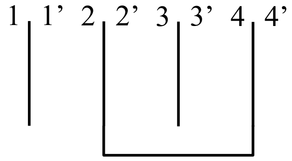
$\Rightarrow C = \frac{\varepsilon_0 q^2}{3d} = \frac{C_0}{3}$

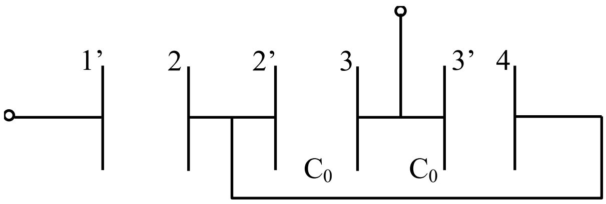
(Q)



$\Rightarrow \left(\frac{C_0}{2}\right)$

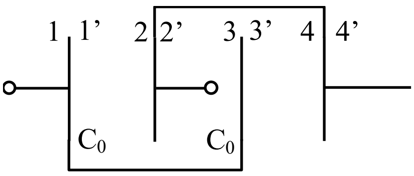
(R)

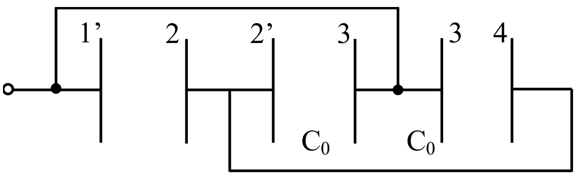


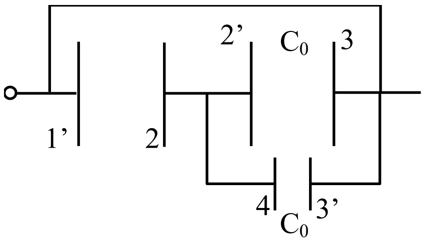


$\Rightarrow \frac{(2C_0)(C_0)}{3C_0} \Rightarrow \frac{2C_0}{3}$

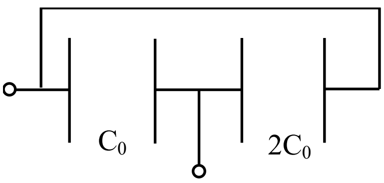
(S)







$\Rightarrow$



$\Rightarrow 3C_0$

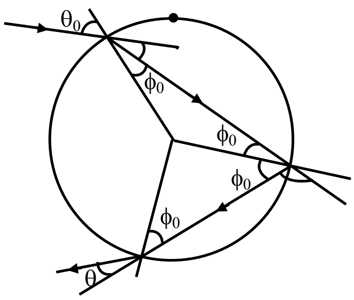
16. A light ray is incident on the surface of a sphere of refractive index n at an angle of incidence θ_0. The ray partially refracts into the sphere with angle of refraction ϕ_0 and then partly reflects from the back surface. The reflected ray then emerges out of the sphere after a partial refraction. The total angle of deviation of the emergent ray with respect to the incident ray is α. Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

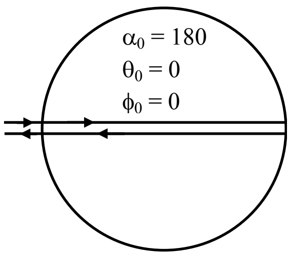
List-I		**List-II**
(P)	If $n = 2$ and $\alpha = 180°$, then all the possible values of θ_0 will be	(1) 30° and 0°
(Q)	If $n = \sqrt{3}$ and $\alpha = 180°$, then all the possible values of θ_0 will be	(2) 60° and 0°
(R)	If $n = \sqrt{3}$ and $\alpha = 180°$, then all the possible values of ϕ_0 will be	(3) 45° and 0°
(S)	If $n = \sqrt{2}$ and $\theta_0 = 45°$, then all the possible values of α will be	(4) 150°
		(5) 0°

(A) P → 5; Q → 2; R → 1; S → 4
(B) P → 5; Q → 1; R → 2; S → 4
(C) P → 3; Q → 2; R → 1; S → 4
(D) P → 3; Q → 1; R → 2; S → 5

Ans. **(A)**

Sol.





$\alpha = (\theta_0 - \phi_0) + (180 - 2\phi_0) + (\theta_0 - 2\phi_0)$

$\alpha = 180 + 2\theta_0 - 4\phi_0$

(P) $\alpha = 180 + 2\theta_0 - 4\phi_0$

$180 = 180 + 2\theta_0 - 4\phi_0 \Rightarrow \theta_0 = 2\phi_0$ ....(i)

$\sin\theta_0 = 2\sin\phi_0$ ....(ii)

From (i) & (ii)

$\sin\theta_0 = 2\sin(\theta_0/2) \Rightarrow \cos\left(\frac{\theta_0}{2}\right) = 1$

$\frac{\theta_0}{2} = 0$

$\Rightarrow \theta_0 = 0$

(Q) $\theta_0 = 2\phi_0$ ....(i)

$\sin\theta_0 = \sqrt{3}\sin\phi_0$ ....(ii)

From (i) & (ii)

$$\sin\theta_0 = \sqrt{3}\sin\left(\frac{\theta_0}{2}\right)$$

$$\Rightarrow \cos\left(\frac{\theta_0}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\frac{\theta_0}{2} = 30,150$$

$\theta_0 = 60$, 300 (Rejected)

$\theta_0 = 60$ & 0

(R) $\theta_0 = 2\phi_0$

$$\sin\theta_0 = \sqrt{3}\sin\phi_0$$

$$\sin 2\theta_0 = \sqrt{3}\sin\phi_0$$

$$\cos\phi_0 = \frac{\sqrt{3}}{2}$$

$\phi_0 = 30$, 150 (Rejected)

$\phi_0 = 30$ & 0 ....(i)

(S) $\sin 45 = \sqrt{2}\cos\phi_0$

$\cos\phi_0 = 1/2$

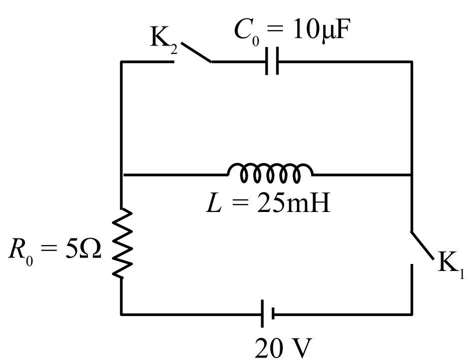
$\phi_0 = 60$

$\alpha = 180 + 2\theta_0 - 4\phi_0$

$\alpha = 180 + 90 - 120$ ....(iv)

$= 180 - 30$; $\alpha = 150°$

17. The circuit shown in the figure contains an inductor L, a capacitor C_0, a resistor R_0 and an ideal battery. The circuit also contains two keys K_1 and K_2. Initially, both the keys are open and there is no charge on the capacitor. At an instant, key K_1 is closed and immediately after this the current in R_0 is found to be I_1. After a long time, the current attains a steady state value I_2. Thereafter, K_2 is closed and simultaneously K_1 is opened and the voltage across C_0 oscillates with amplitude V_0 and angular frequency ω_0.



Match the quantities mentioned in List-I with their values in List-II and choose the correct option.

List-I	List-II
(P) The value of I_1 in Ampere is	(1) 0
(Q) The value of I_2 in Ampere is	(2) 2
(R) The value of ω_0 in kilo-radians/s is	(3) 4
(S) The value of V_0 in Volt is	(4) 20
	(5) 200

(A) P $\rightarrow$ 1; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 5

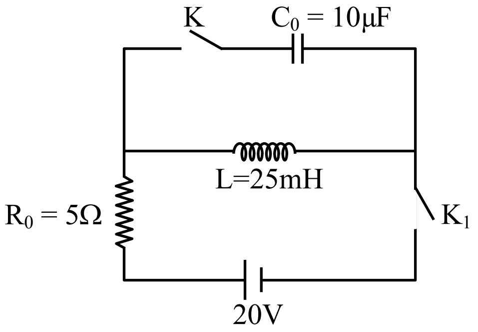
(B) P $\rightarrow$ 1; Q $\rightarrow$ 2; R $\rightarrow$ 3; S $\rightarrow$ 5

(C) P $\rightarrow$ 1; Q $\rightarrow$ 3; R $\rightarrow$ 2; S $\rightarrow$ 4

(D) P $\rightarrow$ 2; Q $\rightarrow$ 5; R $\rightarrow$ 3; S $\rightarrow$ 4

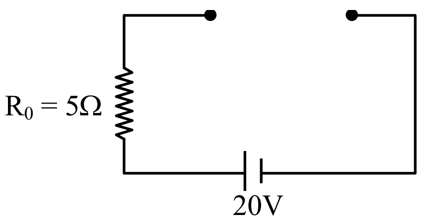
Ans. **(A)**

Sol.



(P) When K_1 is closed current in R_0 is I_1

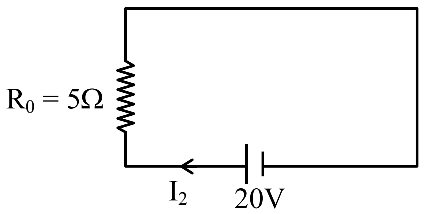
At t = 0; circuit will be



$I_1 = 0$

P $\rightarrow$ (1)

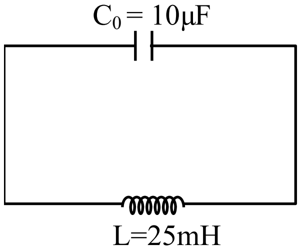
(Q) After long time inductor behave as a wire so I_2



$$I_2 = \frac{20}{5} = 4A$$

Q $\rightarrow$ (3)

(R) When K_2 is closed and K_1 open



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

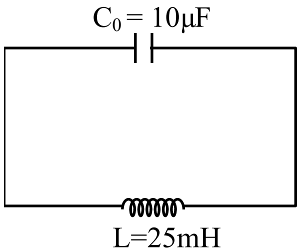
$$\omega_0 = \frac{1}{\sqrt{25\times10^{-3}\times10\times10^{-6}}} = \frac{1}{5\times10^{-4}}$$

$\omega_0 = 2 \times 10^3$ rad/s

$\omega_0 = 2$ kilo-radian/s

$R \rightarrow (2)$

(S) Now K_2 is closed and K_1 open



$$\frac{1}{2}LI_2^2 = \frac{1}{2}CV_0^2$$

$25 \times 10^{-3} \times (4)^2 = 10 \times 10^{-6} \times V_0^2$

$V_0^2 = 2500 \times 16$

$V_0 = 50 \times 4 = 200$ V

$S \rightarrow (5)$