JEE(ADVANCED)–2024 (EXAMINATION)

(Held On Sunday 26th MAY, 2024)

MATHEMATICS | **TEST PAPER WITH ANSWER AND SOLUTION**

PAPER-1

SECTION-1 : (Maximum Marks : 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 If **ONLY** the correct option is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : −1 In all other cases.

---

1. Let $f(x)$ be a continuously differentiable function on the interval $(0, \infty)$ such that $f(1) = 2$ and $\lim\limits_{t\to x}\frac{t^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$ for each $x > 0$. Then, for all $x > 0$, $f(x)$ is equal to :

(A) $\frac{31}{11x}-\frac{9}{11}x^{10}$ (B) $\frac{9}{11x}+\frac{13}{11}x^{10}$ (C) $\frac{-9}{11x}+\frac{31}{11}x^{10}$ (D) $\frac{13}{11x}+\frac{9}{11}x^{10}$

Ans. **(B)**

Sol. $\lim\limits_{t\to x}\frac{t^{10}f(x)-x^{10}f(t)}{t^9-x^9}=1$

$\Rightarrow \lim\limits_{t\to x}\frac{10t^9f(x)-x^{10}f'(t)}{9t^8}=1$

$\Rightarrow 10xf(x) - x^2f'(x) = 9$

$\Rightarrow x^2f'(x) = 10x\, f(x) - 9$

$\Rightarrow f'(x)=\frac{10f(x)}{x}-\frac{9}{x^2}$

$\Rightarrow \frac{dy}{dx}-\frac{10}{x}y=-\frac{9}{x^2}$

$\Rightarrow y.\frac{1}{x^{10}}=\int -\frac{9}{x^2}.\frac{1}{x^{10}}dx$

$\Rightarrow \frac{y}{x^{10}}=\frac{9}{11x^{11}}+c$ ...(1)

$\because f(1)=2\Rightarrow\frac{2}{1}=\frac{9}{11}+c\Rightarrow c=\frac{13}{11}$

$\therefore f(x)=\frac{9}{11x}+\frac{13}{11}x^{10}$

$\Rightarrow$ Option (B) is correct.

2. A student appears for a quiz consisting of only true-false type questions and answers all the questions. The student knows the answers of some questions and guesses the answers for the remaining questions. Whenever the student knows the answer of a question, he gives the correct answer. Assume that the probability of the student giving the correct answer for a question, given that he has guessed it, is $\frac{1}{2}$. Also assume that the probability of the answer for a question being guessed, given that the student's answer is correct, is $\frac{1}{6}$. Then the probability that the student knows the answer of a randomly chosen question is :

(A) $\frac{1}{12}$ (B) $\frac{1}{7}$ (C) $\frac{5}{7}$ (D) $\frac{5}{12}$

Ans. **(C)**

Sol. $C \to$ Correct

$G \to$ Guess

$K \to$ Knows

$P\left(\frac{C}{G}\right)=\frac{1}{2}$, $P\left(\frac{C}{K}\right)=1$

$P\left(\frac{G}{C}\right)=\frac{1}{6}$

Let required probability = x

$$\therefore\ P\left(\frac{G}{C}\right)=\frac{(1-x)P\left(\frac{C}{G}\right)}{(1-x)P\left(\frac{C}{G}\right)+x.P\left(\frac{C}{K}\right)}$$

$$\frac{1}{6}=\frac{(1-x)\left(\frac{1}{2}\right)}{(1-x)\left(\frac{1}{2}\right)+(x)(1)}$$

$\Rightarrow x=\frac{5}{7}$ $\Rightarrow$ Option (C) is correct.

3. Let $\frac{\pi}{2}<x<\pi$ be such that $\cot x=\frac{-5}{\sqrt{11}}$. Then

$$\left(\sin\frac{11x}{2}\right)(\sin 6x-\cos 6x)+\left(\cos\frac{11x}{2}\right)(\sin 6x+\cos 6x)$$

is equal to :

(A) $\frac{\sqrt{11}-1}{2\sqrt{3}}$ (B) $\frac{\sqrt{11}+1}{2\sqrt{3}}$ (C) $\frac{\sqrt{11}+1}{3\sqrt{2}}$ (D) $\frac{\sqrt{11}-1}{3\sqrt{2}}$

Ans. **(B)**

Sol. $x \in \left(\frac{\pi}{2}, \pi\right)$

$\cot x = -\frac{5}{\sqrt{11}}$

$\frac{1-\tan^2\frac{x}{2}}{2\tan\frac{x}{2}} = -\frac{5}{\sqrt{11}}$

$\tan\frac{x}{2} = \sqrt{11}, -\frac{1}{\sqrt{11}}$

$\tan\frac{x}{2} = \sqrt{11}$, As $\frac{\pi}{4} < \frac{x}{2} < \frac{\pi}{2}$

$$\left(\sin\frac{11x}{2}\right)(\sin 6x - \cos 6x) + \left(\cos\frac{11x}{2}\right)(\sin 6x + \cos 6x)$$

$$= \left\{\sin 6x \sin\frac{11x}{2} + \cos\frac{11x}{2}\cos 6x\right\}$$

$$= \cos\left(6x - \frac{11x}{2}\right) + \sin\left(6x - \frac{11x}{2}\right)$$

$$= \cos\frac{x}{2} + \sin\frac{x}{2}$$

$$= \frac{1}{2\sqrt{3}} + \frac{\sqrt{11}}{2\sqrt{3}}$$

$$= \frac{\sqrt{11}+1}{2\sqrt{3}}$$ $\Rightarrow$ Option (B) is correct.

4. Consider the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. Let S(p, q) be a point in the first quadrant such that $\frac{p^2}{9} + \frac{q^2}{4} > 1$. Two tangents are drawn from S to the ellipse, of which one meets the ellipse at one end point of the minor axis and the other meets the ellipse at a point T in the fourth quadrant. Let R be the vertex of the ellipse with positive x-coordinate and O be the center of the ellipse. If the area of the triangle ΔORT is $\frac{3}{2}$, then which of the following options is correct ?

(A) $q = 2,\ p = 3\sqrt{3}$

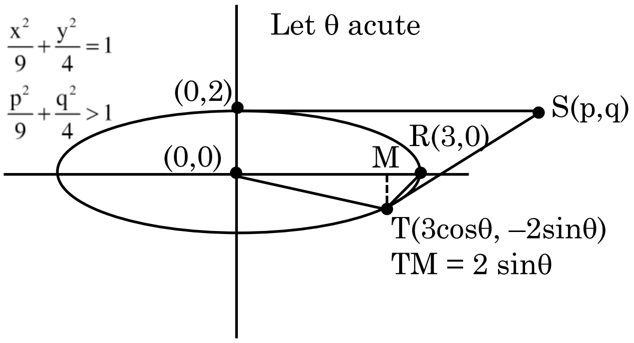
(B) $q = 2,\ p = 4\sqrt{3}$

(C) $q = 1,\ p = 5\sqrt{3}$

(D) $q = 1,\ p = 6\sqrt{3}$

Ans. **(A)**

Sol.



$$\text{Ar}(\Delta ORT) = \frac{3}{2}$$

$$\left|\frac{1}{2}\times 3\times 2\sin\theta\right| = \frac{3}{2}$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{11\pi}{6}$$

$$T\left(\frac{3\sqrt{3}}{2}, -1\right)$$

Tanget at (0, 2) $\frac{x(0)}{9} + \frac{y(2)}{4} = 1 \Rightarrow y = 2 \quad \ldots(1)$

Tangent at $\left(\frac{3\sqrt{3}}{2}, -1\right)$ $\frac{x\left(\frac{3\sqrt{3}}{2}\right)}{9} + \frac{y(-1)}{4} = 1 \quad \ldots(2)$

$\therefore$ By solving (1) & (2) $\Rightarrow$ $p = 3\sqrt{3}$, $q = 2$

$\Rightarrow$ Option (A) is Correct.

SECTION-2 : (Maximum Marks : 12)

- This section contains **THREE (03)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;

Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;

Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;

Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

Negative Marks : −2 In all other cases.

- For example, in a question, if (A), (B) and (D) are the ONLY three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 marks;
  choosing ONLY (B) will get +1 marks;
  choosing ONLY (D) will get +1 marks;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –2 marks.

5. Let S = $\left\{a+b\sqrt{2} : a, b \in \mathbb{Z}\right\}$, $T_1 = \left\{\left(-1+\sqrt{2}\right)^n : n \in \mathbb{N}\right\}$ and $T_2 = \left\{\left(1+\sqrt{2}\right)^n : n \in \mathbb{N}\right\}$. Then which of the following statements is (are) TRUE ?

(A) $\mathbb{Z} \bigcup T_1 \bigcup T_2 \subset S$

(B) $T_1 \bigcap \left(0, \frac{1}{2024}\right) = \phi$, where ϕ denotes the empty set.

(C) $T_2 \bigcap \left(2024, \infty\right) \neq \phi$

(D) For any given a, b $\in \mathbb{Z}$, $\cos\left(\pi(a+b\sqrt{2})\right) + i\sin\left(\pi(a+b\sqrt{2})\right) \in \mathbb{Z}$ if and only if b = 0, where $i = \sqrt{-1}$.

Ans. **(A,C,D)**

Sol. (A) $\left(-1+\sqrt{2}\right)^n = m + \sqrt{2}n, m, n \in \mathbb{Z}$

$\left(1+\sqrt{2}\right)^n = m_1 + \sqrt{2}n_1, m_1, n_1 \in \mathbb{Z}$

$\Rightarrow \mathbb{Z} \cup T_1 \cup T_2 \subseteq S$

but $b\sqrt{2} \in S$ for negative $b \in \mathbb{Z}$.

So $\mathbb{Z} \cup T_1 \cup T_2 \subset S$

(B) $\left(\sqrt{2}-1\right)^n = \frac{1}{\left(\sqrt{2}+1\right)^n} < \frac{1}{2024}$

$\Rightarrow 2024 < \left(\sqrt{2}+1\right)^n, \exists n \in \mathbb{N}$

$\Rightarrow T_1 \cap \left(0, \frac{1}{2024}\right) \neq \phi$

(C) $\left(1+\sqrt{2}\right)^n > 2024, \exists n \in \mathbb{N}$

$\Rightarrow T_2 \cap \left(2024, \infty\right) \neq \phi$

(D) $\sin\left(\pi\left(a+b\sqrt{2}\right) = 0\right) \Rightarrow b = 0,\ a \in \mathbb{Z}$.

$\Rightarrow$ Options (A), (C), (D) are Correct.

6. Let $\mathbb{R}^2$ denote $\mathbb{R}\times\mathbb{R}$. Let

S = {(a, b, c) : a, b, c $\in \mathbb{R}$ and $ax^2 + 2bxy + cy^2 > 0$ for all (x, y) $\in \mathbb{R}^2 - \{(0, 0)\}$}.

Then which of the following statements is (are) TRUE ?

(A) $\left(2, \frac{7}{2}, 6\right) \in S$

(B) If $\left(3, b, \frac{1}{12}\right) \in S$, then |2b| < 1.

(C) For any given (a, b, c) $\in$ S, the system of linear equations

$ax + by = 1$

$bx + cy = -1$

has a unique solution.

(D) For any given (a, b, c) $\in$ S, the system of linear equations

$(a + 1)x + by = 0$

$bx + (c + 1)y = 0$

has a unique solution

Ans. **(B,C,D)**

Sol. (A) $ax^2 + 2bxy + cy^2 > 0 \ \forall (x, y) \in \mathbb{R}^2 - \{(0,0)\}$

$\Rightarrow ax^2 + 2bxy + cy^2$ must represent pair of imaginary lines and a, c > 0.

$\Rightarrow b^2 < ac$

(B) $b^2 < 3 \times \frac{1}{12} \Rightarrow |2b| < 1$

(C) since $b^2 \neq ac$

$\Rightarrow$ ax + by = 1 and bx + cy = –1

are not parallel lines.

(D) $ac + a + c > b^2 \Rightarrow$ lines are

not parallel.

$\Rightarrow$ Options (B), (C), (D) are Correct.

7. Let $\mathbb{R}^3$ denote the three-dimensional space. Take two points P = (1, 2, 3) and Q = (4, 2, 7). Let dist (X, Y) denote the distance between two points X and Y in $\mathbb{R}^3$. Let

$S = \left\{X \in \mathbb{R}^3 : (\text{dist}(X, P))^2 - (\text{dist}(X, Q))^2 = 50\right\}$ and

$T = \left\{Y \in \mathbb{R}^3 : (\text{dist}(Y, Q))^2 - (\text{dist}(Y, P))^2 = 50\right\}$.

Then which of the following statements is (are) TRUE ?

(A) There is a triangle whose area is 1 and all of whose vertices are from S.

(B) There are two distinct points L and M in T such that each point on the line segment LM is also in T.

(C) There are infinitely many rectangles of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

(D) There is a square of perimeter 48, two of whose vertices are from S and the other two vertices are from T.

Ans. **(A,B,C,D)**

Sol. $S = \{X : (XP)^2 - (XQ)^2 = 50\}$

$T = \{Y : (YQ)^2 - (YP)^2 = 50\}$

for finding $S \equiv X (x, y, z)$ and for $T \equiv Y(x, y, z)$

$((x-1)^2 + (y-1)^2 + (z-1)^2) - ((x-4)^2 + (y-2)^2 + (z-7)^2) = 50$

$\Rightarrow \quad S = \{(x, y, z) : 6x + 8z = 105\}$

$T = \{(x, y, z) : 6x + 8z = 5\}$

Since S and T both are plane ;

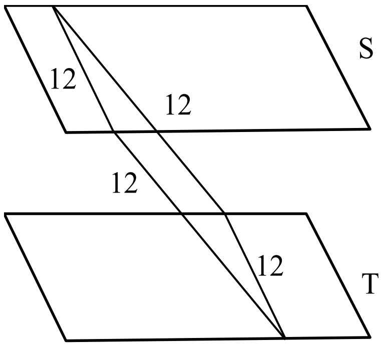
(A) There exist a triangle in plane S whose area = 1 (always)

(B) L & M lies on plane T, hence line segment joining L & M will lie on plane T.

(C) Distance between S & T

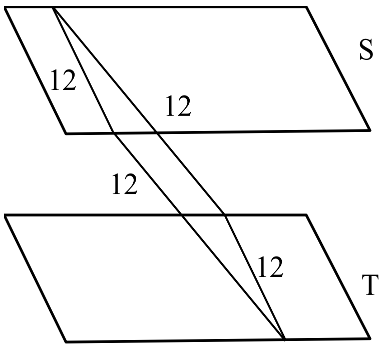
$$d = \left|\frac{105-5}{10}\right| = 10$$

Hence for rectangle of perimeter 48 can exist.



There will be infinite such rectangle possible.

(D) For square



Hence Answers A,B,C,D are correct

SECTION-3 : (Maximum Marks : 24)

- This section contains **SIX (06)** questions.
- The answer to each question is a **NON-NEGATIVE INTEGER.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.
- Answer to each question will be evaluated according to the following marking scheme:

  Full Marks : +4 **ONLY** If the correct integer is entered;

  Zero Marks : 0 In all other cases.

---

8. Let $a = 3\sqrt{2}$ and $b = \dfrac{1}{5^{1/6}\sqrt{6}}$. If $x, y \in \mathbb{R}$ are such that

$3x + 2y = \log_a (18)^{\frac{5}{4}}$ and

$2x - y = \log_b (\sqrt{1080})$,

then $4x + 5y$ is equal to ……..

Ans. **(8)**

Sol. $3x + 2y = \log_{3\sqrt{2}} \left(3\sqrt{2}\right)^{\frac{5}{2}} = \dfrac{5}{2}$

$\Rightarrow \quad 6x + 4y = 5 \quad ……(1)$

$2x - y = \log_{\frac{1}{5^{1/6}\sqrt{6}}} (5^{\frac{1}{6}}\sqrt{6})^3 = -3$

$\Rightarrow \quad 2x - y = -3 \quad …..(2)$

equation (1) – (2)

$\Rightarrow \quad 4x + 5y = 8$

9. Let $f(x) = x^4 + ax^3 + bx^2 + c$ be a polynomial with real coefficients such that $f(1) = -9$. Suppose that $i\sqrt{3}$ is a root of the equation $4x^3 + 3ax^2 + 2bx = 0$, where $i = \sqrt{-1}$. If α_1, α_2, α_3, and α_4 are all the roots of the equation $f(x) = 0$, then $|\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2$ is equal to ………

Ans. **(20)**

Sol. $f(1) = 1 + a + b + c = -9 \quad \Rightarrow \quad a + b + c = -10 \quad …..(1)$

$4x^3 + 3ax^2 + 2bx = 0$ roots are $\sqrt{3}\,i,\ -\sqrt{3}\,i,\ 0$

$\Rightarrow \quad 4x^2 + 3ax + 2b = 0$ with roots $\sqrt{3}\,i$ and $-\sqrt{3}\,i$

$\Rightarrow \quad a = 0 \ \&\ \dfrac{2b}{4} = \left(\sqrt{3}\,i\right)\left(-\sqrt{3}\,i\right)$

$b = 6$ use a, b in (1) $\Rightarrow c = -16$

$\Rightarrow \quad f(x) = x^4 + 6x^2 - 16 = 0$

$(x^2 + 8)(x^2 - 2) = 0$

$\Rightarrow \quad x = \pm\sqrt{8}\,i, \pm\sqrt{2} \quad \Rightarrow \quad |\alpha_1|^2 + |\alpha_2|^2 + |\alpha_3|^2 + |\alpha_4|^2 = 20$

10. Let $S = \left\{ A = \begin{pmatrix} 0 & 1 & c \\ 1 & a & d \\ 1 & b & e \end{pmatrix} : a, b, c, d, e \in \{0,1\} \text{ and } |A| \in \{-1, 1\} \right\}$, where |A| denotes the determinant of A. Then the number of elements in S is ______.

Ans. **(16)**

10. $|A| = 0(ae - bd) - 1(e - d) + c(b - a)$

$= c(b - a) + (d - e)$

$|A| \in \{-1, 1\}$ and $a, b, c, d, e \in \{0, 1\}$

Case-I

$c = 0 \quad d = 1, e = 0, a, b \in (0, 1)$

$d = 0, e = 1$

a b c d e

↓

2 2 1 2 → 8 cases

Case-II

$c = 1 \quad b = 1, a = 0, \quad d = 0, e = 0, d = 1, e = 1$

$b = 0, a = 1, \quad d = 0, e = 0, d = 1, e = 1$

$b = 0, a = 0, \quad d = 1, e = 0$

$d = 0, e = 1$

$b = 1, a = 1, \quad d = 1, e = 0$

$d = 0, e = 1$

→8 cases

⇒ Total 16 cases

11. A group of 9 students, $s_1, s_2, \ldots\ldots, s_9$, is to be divided to from three teams X, Y, and Z of sizes 2, 3, and 4, respectively. Suppose that s_1 cannot be selected for the team X, and s_2 cannot be selected for the team Y. Then the number of ways to from such teams, is ______.

Ans. **(665)**

Sol. x y z

2 3 4

$\overline{S}_1$ $\overline{S}_2$

C-i) when x does not contain S_1, but contains S_2

$$\underset{\text{for x}}{{}^7C_1} \times \underset{\text{for y,z}}{\frac{7!}{3!4!}} = 245$$

C-ii) When x does not contain S_1, S_2 and y does not contain S_2

$$\text{i.e. } \underset{\text{for x}}{{}^7C_2} \times \underset{\text{for y,z}}{\frac{6!}{3!3!}} = 420$$

so total No. of ways 665

12. Let $\overrightarrow{OP} = \frac{\alpha-1}{\alpha}\hat{i} + \hat{j} + \hat{k}$, $\overrightarrow{OQ} = \hat{i} + \frac{\beta-1}{\beta}\hat{j} + \hat{k}$ and $\overrightarrow{OR} = \hat{i} + \hat{j} + \frac{1}{2}\hat{k}$ be three vectors, where $\alpha, \beta \in \mathbb{R} - \{0\}$ and O denotes the origin. If $\left(\overrightarrow{OP} \times \overrightarrow{OQ}\right).\overrightarrow{OR} = 0$ and the point (α, β, 2) lies on the plane $3x + 3y - z + l = 0$, then the value of l is ……….

Ans. **(5)**

Sol. $\left(\overrightarrow{OP} \times \overrightarrow{OQ}\right).\overrightarrow{OR} = 0$

$$\begin{vmatrix} \frac{\alpha-1}{\alpha} & 1 & 1 \\ 1 & \frac{\beta-1}{\beta} & 1 \\ 1 & 1 & \frac{1}{2} \end{vmatrix} = 0$$

$\alpha + \beta + 1 = 0$ …(1)

Also (α, β, 2) lies on $3x + 3y - z + l = 0$

$\Rightarrow$ $3\alpha + 3\beta - 2 + l = 0$ $\Rightarrow$ $l = 2 - 3(\alpha + \beta)$

use (1) in it $\Rightarrow l = 5$

13. Let X be a random variable, and let P(X = x) denote the probability that X takes the value x. Suppose that the points (x, P(X = x)), x = 0, 1, 2, 3, 4, lie on a fixed straight line in the xy-plane, and P(X = x) = 0 for all $x \in \mathbb{R} - \{0, 1, 2, 3, 4\}$. If the mean of X is $\frac{5}{2}$, and the variance of X is α, then the value of 24α is ……..

Ans. **(42)**

Sol. Let equation of line is y = mx + c

x	0	1	2	3	4	R–{0,1,2,3,4}
P(x)	c	m + c	2m + c	3m + c	4m + c	0

$\sum_{x=0}^{4}(mx + c) = 1 \Rightarrow 10m + 5c = 1 \Rightarrow 2m + c = \frac{1}{5}$ …(1)

mean $= \Sigma x_i P_i = \sum_{i=0}^{4}(mx_i + c).x_i = 30m + 10c = \frac{5}{2}$

$\therefore 3m + c = \frac{1}{4}$ …(2)

from (1) and (2) $m = \frac{1}{20}$, $c = \frac{1}{10}$

$\Sigma P_i x_i^2 = \sum_{i=0}^{4}\left(mx_i + c\right)x_1^2$

$= \sum_{i=0}^{4}\left(mx_i^3 + cx_i^2\right) \Rightarrow 100m + 30c$ (Now putting m and c)

$\Rightarrow \Sigma P_i x_i^2 = 5 + 3 = 8$

Variance $= \Sigma P_i x_i^2 - \left(\Sigma P_i x_i\right)^2 = 8 - \left(\frac{5}{2}\right)^2 = \frac{7}{4}$

$\therefore 24\alpha = 42$

SECTION-4 : (Maximum Marks : 12)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists : **List-I** and **List-II**.
- **List-I** has **Four** entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:

Full Marks : +3 **ONLY** if the option corresponding to the correct combination is chosen;
Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
Negative Marks : –1 In all other cases.

---

14. Let α and β be the distinct roots of the equation $x^2 + x - 1 = 0$. Consider the set $T = \{1, \alpha, \beta\}$. For a 3×3 matrix $M = (a_{ij})_{3\times3}$, define $R_i = a_{i1} + a_{i2} + a_{i3}$ and $C_j = a_{1j} + a_{2j} + a_{3j}$ for i = 1, 2, 3 and j = 1, 2, 3.

Match each entry in **List-I** to the correct entry in **List-II**.

List-I		**List-II**	
(P)	The number of matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $R_i = C_j = 0$ for all i, j, is	(1)	1
(Q)	The number of symmetric matrices $M = (a_{ij})_{3\times3}$ with all entries in T such that $C_j = 0$ for all j, is	(2)	12
(R)	Let $M = (a_{ij})_{3\times3}$ be a skew symmetric matrix such that $a_{ij} \in T$ for $i > j$. Then the number of elements in the set $\left\{\begin{pmatrix} x \\ y \\ z \end{pmatrix} : x, y, z \in \mathbb{R}, M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_{12} \\ 0 \\ -a_{23} \end{pmatrix}\right\}$ is	(3)	infinite
(S)	Let $M = (a_{ij})_{3\times3}$ be a matrix with all entries in T such that $R_i = 0$ for all i. Then the absolute value of the determinant of M is	(4)	6
		(5)	0

The correct options is

(A) (P) $\to$ (4) (Q) $\to$ (2) (R) $\to$ (5) (S) $\to$ (1)

(B) (P) $\to$ (2) (Q) $\to$ (4) (R) $\to$ (1) (S) $\to$ (5)

(C) (P) $\to$ (2) (Q) $\to$ (4) (R) $\to$ (3) (S) $\to$ (5)

(D) (P) $\to$ (1) (Q) $\to$ (5) (R) $\to$ (3) (S) $\to$ (4)

Ans. **(C)**

Sol. α, β are roots of $x^2 + x - 1 = 0$

$\therefore \alpha + \beta = -1 \Rightarrow 1 + \alpha + \beta = 0$

$$M = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

(P) $$M = \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ \beta & 1 & \alpha \end{bmatrix} \Rightarrow 3! \times 2 = 12$$

For one arrangement of row 1 we can arrange other two rows exactly in two ways and row 1 can be arranged in 3! ways

$\therefore 3! \times 2 = 12$ ways

(Q) $$M = \begin{bmatrix} x & a & b \\ a & y & c \\ b & c & z \end{bmatrix} \Rightarrow \text{Consider one such arrangement with } a = \alpha, b = \beta, c = 1$$

$$M = \begin{bmatrix} 1 & \alpha & \beta \\ \alpha & \beta & 1 \\ \beta & 1 & \alpha \end{bmatrix}$$

a, b, c can be arranged in 3! ways and corresponding entries can be arranged in 1 way.

(R) $$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ 0 \\ -c \end{bmatrix}$$

$ay + bz = a$

$-ax + cz = 0$

$-bx - cy = -c$

It is observed that $D = D_x = D_y = D_z = 0$

$\therefore$ infinite solution

(S) $$\begin{bmatrix} 1 & \alpha & \beta \\ \beta & \alpha & 1 \\ \alpha & 1 & \beta \end{bmatrix}$$

$\Rightarrow \alpha\beta - 1 - \alpha\beta^2 + \alpha^2 + \beta^2 - \alpha^2\beta = 0$ (since $\alpha\beta = \alpha + \beta = -1$)

15. Let the straight line $y = 2x$ touch a circle with center $(0, \alpha)$, $\alpha > 0$, and radius r at a point A_1. Let B_1 be the point on the circle such that the line segment A_1B_1 is a diameter of the circle. Let $\alpha + r = 5 + \sqrt{5}$.

Match each entry in **List-I** to the correct entry in **List-II**.

List-I		List-II	
(P)	α equals	(1)	$(-2, 4)$
(Q)	r equals	(2)	$\sqrt{5}$
(R)	A_1 equals	(3)	$(-2, 6)$
(S)	B_1 equals	(4)	5
		(5)	$(2, 4)$

The correct option is

(A) (P) $\to$ (4) (Q) $\to$ (2) (R) $\to$ (1) (S) $\to$ (3)

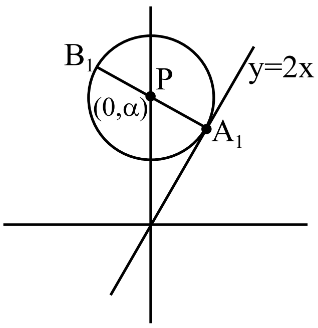
(B) (P) $\to$ (2) (Q) $\to$ (4) (R) $\to$ (1) (S) $\to$ (3)

(C) (P) $\to$ (4) (Q) $\to$ (2) (R) $\to$ (5) (S) $\to$ (3)

(D) (P) $\to$ (2) (Q) $\to$ (4) (R) $\to$ (3) (S) $\to$ (5)

Ans. **(C)**

Sol.



Consider centre as $P(0, \alpha)$, $\alpha > 0$

$$\left|\frac{2(0) - \alpha}{\sqrt{5}}\right| = r$$

$|-\alpha| = \sqrt{5}\,r$

$\alpha = \sqrt{5}\,r$

$\therefore\ \alpha + r = 5 + \sqrt{5}$

$\sqrt{5}\,r + r = \sqrt{5}\left(\sqrt{5}+1\right)$

$r = \sqrt{5},\ \ \alpha = 5$

$\therefore\ P(0, 5)$

Foot of perpendicular from P to line $2x - y = 0$

$$\frac{x-0}{2} = \frac{y-5}{-1} = \frac{-(2(0)-5)}{5} = 1$$

$x = 2,\ \ y = 4 \quad A_1(2, 4)$

Let $B(p, q)\quad \therefore\ \dfrac{p+2}{2} = 0,\ \dfrac{q+4}{2} = 5$

$\therefore\ p = -2,\ q = 6\quad B(-2, 6)$

16. Let $\gamma \in \mathbb{R}$ be such that the lines $L_1 : \dfrac{x+11}{1} = \dfrac{y+21}{2} = \dfrac{z+29}{3}$ and $L_2 : \dfrac{x+16}{3} = \dfrac{y+11}{2} = \dfrac{z+4}{\gamma}$ intersect.

Let R_1 be the point of intersection of L_1 and L_2. Let $O = (0, 0, 0)$, and $\hat{n}$ denote a unit normal vector to the plane containing both the lines L_1 and L_2.

Match each entry in **List-I** to the correct entry in **List-II**.

List-I		**List-II**	
(P)	γ equals	(1)	$-\hat{i} - \hat{j} + \hat{k}$
(Q)	A possible choice for $\hat{n}$ is	(2)	$\sqrt{\dfrac{3}{2}}$
(R)	$\overrightarrow{OR_1}$ equals	(3)	1
(S)	A possible value of $\overrightarrow{OR_1}.\hat{n}$ is	(4)	$\dfrac{1}{\sqrt{6}}\hat{i} - \dfrac{2}{\sqrt{6}}\hat{j} + \dfrac{1}{\sqrt{6}}\hat{k}$
		(5)	$\sqrt{\dfrac{2}{3}}$

The correct option is

(A) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (4) (R) $\rightarrow$ (1) (S) $\rightarrow$ (2)

(B) (P) $\rightarrow$ (5) (Q) $\rightarrow$ (4) (R) $\rightarrow$ (1) (S) $\rightarrow$ (2)

(C) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (4) (R) $\rightarrow$ (1) (S) $\rightarrow$ (5)

(D) (P) $\rightarrow$ (3) (Q) $\rightarrow$ (1) (R) $\rightarrow$ (4) (S) $\rightarrow$ (5)

Ans. **(C)**

Sol. $L_1 : \frac{x+11}{1} = \frac{y+21}{2} = \frac{z+29}{3} = a$

$L_2 : \frac{x+16}{3} = \frac{y+11}{2} = \frac{z+4}{\gamma} = b$

$x = a - 11 = 3b - 16 \Rightarrow a - 3b = -5$ ...(1)

$y = 2a - 21 = 2b - 11 \Rightarrow 2a - 2b = 10$ ...(2)

$z = 3a - 29 = br - 4 \Rightarrow 3a - b\gamma = 25$ ...(3)

from (1) & (2)

$a = 10$, $b = 5$

Now from (3)

$3(10) - 5\gamma = 25$ $\therefore \gamma = 1$

$R_1 \equiv (-1, -1, 1)$

$OR_1 = -\hat{i} - \hat{j} + \hat{k}$

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = -4\hat{i} - (-8)\hat{j} - 4\hat{k}$$

$$\vec{n} = -4\hat{i} + 8\hat{j} + 4\hat{k} = -4(\hat{i} - 2\hat{j} + \hat{k})$$

$$\hat{n} = \pm\frac{4(\hat{i} - 2\hat{j} + \hat{k})}{4\sqrt{6}} = \pm\frac{(\hat{i} - 2\hat{j} + \hat{k})}{\sqrt{6}}$$

$$\overrightarrow{OR} \cdot \hat{n} = \pm\left(-\hat{i} - \hat{j} + \hat{k}\right)\left(\frac{\hat{i} - 2\hat{j} + \hat{k}}{\sqrt{6}}\right) = \pm\frac{2}{\sqrt{6}} = \pm\sqrt{\frac{4}{6}} = \pm\sqrt{\frac{2}{3}}$$

17. Let $f:\mathbb{R}\rightarrow\mathbb{R}$ and $g:\mathbb{R}\rightarrow\mathbb{R}$ be functions defined by

$$f(x)=\begin{cases} x|x|\sin\left(\frac{1}{x}\right), & x\neq 0,\\ 0, & x=0,\end{cases} \quad \text{and} \quad g(x)=\begin{cases} 1-2x, & 0\leq x\leq \frac{1}{2},\\ 0, & \text{otherwise.}\end{cases}$$

Let a, b, c, d $\in\mathbb{R}$. Define the function $h:\mathbb{R}\rightarrow\mathbb{R}$ by

$$h(x)=a f(x)+b\left(g(x)+g\left(\frac{1}{2}-x\right)\right)+c(x-g(x))+d\,g(x),\ x\in\mathbb{R}$$

Match each entry in **List-I** to the correct entry in **List-II**.

List-I		**List-II**	
(P)	If a = 0, b = 1, c = 0 and d = 0, then	(1)	h is one-one.
(Q)	If a = 1, b = 0, c = 0 and d = 0, then	(2)	h is onto.
(R)	If a = 0, b = 0, c = 1 and d = 0, then	(3)	h is differentiable on $\mathbb{R}$.
(S)	If a = 0, b = 0, c = 0 and d = 1, then	(4)	the range of h is [0, 1].
		(5)	the range of h is {0, 1}.

The correct option is :

(A) (P) → (4) (Q) → (3) (R) → (1) (S) → (2)

(B) (P) → (5) (Q) → (2) (R) → (4) (S) → (3)

(C) (P) → (5) (Q) → (3) (R) → (2) (S) → (4)

(D) (P) → (4) (Q) → (2) (R) → (1) (S) → (3)

Ans. **(C)**

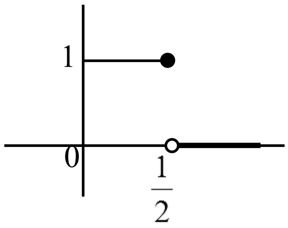
Sol. $$f(x)=\begin{cases} x|x|\sin\frac{1}{x} & ; \ x\neq 0\\ 0 & ; \ x=0\end{cases} \qquad g(x)=\begin{cases} 1-2x & ; \ 0\leq x\leq\frac{1}{2}\\ 0 & ; \ \text{otherwise}\end{cases}$$

$$g\left(\frac{1}{2}-x\right)=\begin{cases} 2x & ; \ 0\leq\frac{1}{2}-x\leq\frac{1}{2}\\ 0 & ; \ \text{otherwise}\end{cases}=\begin{cases} 2x & ; \ 0\leq x\leq\frac{1}{2}\\ 0 & ; \ \text{otherwise}\end{cases}$$

$$g(x)+g\left(\frac{1}{2}-x\right)=\begin{cases} 1 & ; \ 0\leq x\leq\frac{1}{2}\\ 0 & ; \ \text{otherwise}\end{cases}$$

(P) Now a = 0, b = 1, c = 0, d = 0

$$\because\ h(x)=g(x)+g\left(\frac{1}{2}-x\right)=\begin{cases}1 & ; & 0\le x\le \frac{1}{2}\\ 0 & ; & \text{otherwise}\end{cases}$$



Hence Range of h(x) is {0, 1}

(Q) a = 1, b = 0, c = 0, d = 0

$$h(x)=f(x)=\begin{cases}x|x|\sin\frac{1}{x} & ; & x\ne 0\\ 0 & ; & x=0\end{cases}$$

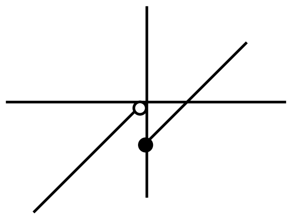
$$\text{RHD}=\lim_{x\to 0}\frac{x^2\sin\frac{1}{x}-0}{x}=0$$

$$\text{LHD}=\lim_{x\to 0}\frac{-x^2\sin\frac{1}{x}-0}{x}=0$$

Hence h(x) is differentiable on R

(R) a = 0, b = 0, c = 1, d = 0

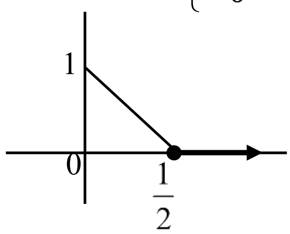
$$h(x)=x-g(x)=\begin{cases}3x-1 & ; & 0\le x\le \frac{1}{2}\\ 0 & ; & \text{otherwise}\end{cases}$$



$\therefore$ h(x) is ONTO

(S) a = 0, b = 0, c = 0, d = 1

$$h(x)=g(x)=\begin{cases}1-2x & ; & 0\le x\le \frac{1}{2}\\ 0 & ; & \text{otherwise}\end{cases}$$



Range of h(x) is [0, 1]