JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON THURSDAY 23rd JANUARY 2025) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. If in the expansion of $(1 + x)^p (1 - x)^q$, the coefficients of x and x^2 are 1 and –2, respectively, then $p^2 + q^2$ is equal to :

(1) 8 (2) 18

(3) 13 (4) 20

Ans. (3)

Sol. $(1+x)^p(1-x)^q = \left({}^pC_0 + {}^pC_1x + {}^pC_2x^2 + \ldots\right)\left({}^qC_0 - {}^qC_1x + {}^qC_2x^2 + \ldots\right)$

coff of $x \equiv {}^pC_0\,{}^qC_1 + {}^pC_1\,{}^qC_0 = 1$

$p - q = 1$

coff of $x^2 \equiv {}^pC_0\,{}^qC_2 - {}^pC_1\,{}^qC_1 + {}^pC_2\,{}^qC_0 = -2$

$$\frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2} = -2$$

$q^2 - q - 2pq + p^2 - p = -4$

$1 - (p + q) = -4$

$p + q = 5$

$p = 3$

$q = 2$

so $p^2 + q^2 = 13$

2. Let $A = \{(x, y) \in \mathbf{R} \times \mathbf{R} : |x + y| \geq 3\}$ and $B = \{(x, y) \in \mathbf{R} \times \mathbf{R} : |x| + |y| \leq 3\}$.

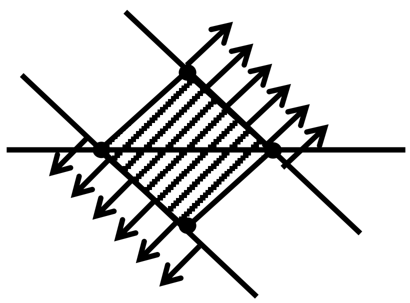
If $C = \{(x, y) \in \mathbf{A} \cap \mathbf{B} : x = 0 \text{ or } y = 0\}$, then $\sum_{(x,y)\in C} |x+y|$ is :

(1) 15 (2) 18

(3) 24 (4) 12

Ans. (4)

Sol.



$C = \{(3,0), (-3,0), (0,3), (0,-3)\}$

$\Sigma|x + y| = 12$

3. The system of equations

$x + y + z = 6,$

$x + 2y + 5z = 9,$

$x + 5y + \lambda z = \mu,$

has no solution if

(1) $\lambda = 17, \mu \neq 18$

(2) $\lambda \neq 17, \mu \neq 18$

(3) $\lambda = 15, \mu \neq 17$

(4) $\lambda = 17, \mu = 18$

Ans. (1)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 5 & \lambda \end{vmatrix} = 0$

$\lambda = 17$

$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 9 \\ 1 & 5 & \mu \end{vmatrix} \neq 0$

$\mu \neq 18$

4. Let $\int x^3 \sin x\,dx = g(x) + C$, where C is the constant of integration.

If $8\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \alpha\pi^3 + \beta\pi^2 + \gamma$, $\alpha, \beta, \gamma \in Z$,

Then $\alpha + \beta - \gamma$ equals :

(1) 55 (2) 47

(3) 48 (4) 62

Ans. (1)

Sol. $\int x^3 \sin x\,dx = -x^3 \cos x + \int 3x^2 \cos x\,dx$

$= -x^3 \cos x + 3x^2 \sin x - \int 6x \sin x\,dx$

$= -x^3 \cos x + 3x^2 \sin x + 6x\cos x - 6\sin x + c$

So $g(x) = -x^3 \cos x + 3x^2 \sin x + 6x\cos x - 6\sin x$

$$g\left(\frac{\pi}{2}\right) = \frac{3\pi^2}{4} - 6$$

$g'(x) = -3x^2 \cos x + x^3 \sin x + 6\cos x - 6\cos x$

$$g'\left(\frac{\pi}{2}\right) = \frac{\pi^3}{8}$$

$$8\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \pi^3 + 6\pi^2 - 48$$

So $\alpha + \beta - \gamma = 55$

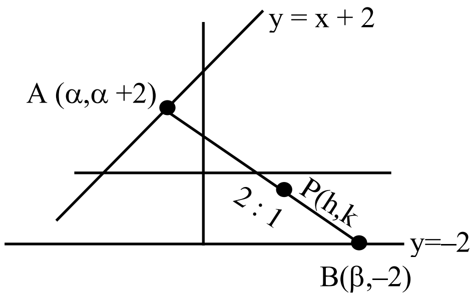
5. A rod of length eight units moves such that its ends A and B always lie on the lines $x - y + 2 = 0$ and $y + 2 = 0$, respectively. If the locus of the point P, that divides the rod AB internally in the ratio 2 : 1 is $9(x^2 + \alpha y^2 + \beta xy + \gamma x + 28y) - 76 = 0$, then $\alpha - \beta - \gamma$ is equal to :

(1) 24 (2) 23

(3) 21 (4) 22

Ans. **(2)**

Sol.



$h = \frac{3\beta + \alpha}{3}$

$k = \frac{-4 + \alpha + 2}{3}$

$\alpha = 3k + 2$

$2\beta = 3h - a = 3h - 3k - 2$

so AB = 8

$(\alpha - \beta)^2 + (\alpha + 4)^2 = 64$

$\left(3k + 2 - \left(\frac{3h - 3k - 2}{2}\right)\right)^2 + (3k + 2 + 4)^2 = 64$

$\frac{(9k - 3h + 6)^2}{4} + (3k + 6)^2 = 64$

$9\left[(3k - h + 2)^2 + 4(k + 2)^2\right] = 64 \times 4$

$9(x^2 + 13y^2 - 6xy - 4x + 28y) = 76$

$\alpha - \beta - \gamma = 13 + 6 + 4 = 23$

6. The distance of the line $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ from the point (1, 4, 0) along the line $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ is :

(1) $\sqrt{17}$ (2) $\sqrt{14}$

(3) $\sqrt{15}$ (4) $\sqrt{13}$

Ans. **(2)**

Sol. Let the parallel line is

$\frac{x-1}{1} = \frac{y-4}{2} = \frac{z-0}{3}$

so their point of intersection is

$(\lambda + 1, 2\lambda + 4\ 3\lambda) = (2t + 2, 3t + 6, 4t + 3)$

$\lambda = 2t + 1$

$2\lambda + 4 = 3t + 6 \Rightarrow t = 0$

so POI is (2,6,3)

so distance $= \sqrt{(2-1)^2 + (6-4)^2 + (3-0)^2} = \sqrt{14}$

7. Let the point A divide the line segment joining the points P(–1, –1, 2) and Q(5, 5,10) internally in the ratio r : 1 (r > 0). If O is the origin and $\left(\overrightarrow{OQ}.\overrightarrow{OA}\right) - \frac{1}{5}\left|\overrightarrow{OP} \times \overrightarrow{OA}\right|^2 = 10$, then the value of r is :

(1) 14 (2) 3

(3) $\sqrt{7}$ (4) 7

Ans. **(4)**

Sol. $A \equiv \left(\frac{5r-1}{r+1}, \frac{5r-1}{r+1}, \frac{10r+2}{r+1}\right)$

$\left(\overrightarrow{OQ}.\overrightarrow{OA}\right) - \frac{\left|\overrightarrow{OP} \times \overrightarrow{OA}\right|^2}{5} = 10$ ....(1)

$\overrightarrow{OQ}.\overrightarrow{OA} = \frac{5}{r+1}(30r + 2)$

$\left|\overrightarrow{OP} \times \overrightarrow{OA}\right|^2 = \frac{r^2}{(r+1)^2}(800)$

so by equation (1)

$\frac{10}{r+1}(15r + 1) - \frac{1}{5}\frac{r^2(800)}{(r+1)^2} = 10$

$2r^2 - 14r = 0$

$r = 7, r \neq 0$

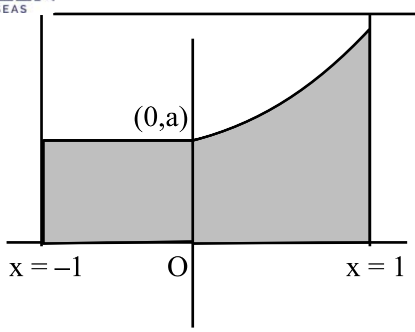
8. If the area of the region $\{(x, y) : -1 \le x \le 1, 0 \le y \le a + e^{|x|} - e^{-x}, a > 0\}$ is $\frac{e^2 + 8e + 1}{e}$, then the value of a is :

(1) 7 (2) 6

(3) 8 (4) 5

Ans. **(4)**

Sol.



required area is $a+\int_0^1\left(a+e^x-e^{-x}\right)dx$

$a+\left[a+e^x+e^{-x}\right]_0^1$

$2a+e-1+e^{-1}-1=e+8+\frac{1}{e}$

$2a=10\Rightarrow a=5$

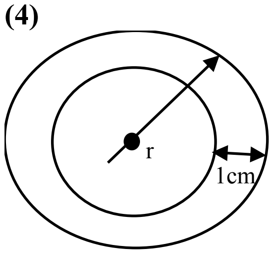
9. A spherical chocolate ball has a layer of ice-cream of uniform thickness around it. When the thickness of the ice-cream layer is 1 cm, the ice-cream melts at the rate of 81 cm^3/min and the thickness of the ice-cream layer decreases at the rate of $\frac{1}{4\pi}$ cm/min. The surface area (in cm^2) of the chocolate ball (without the ice-cream layer) is :

(1) $225\,\pi$ (2) $128\,\pi$

(3) $196\,\pi$ (4) $256\,\pi$

Ans. (4)

Sol



$v=\frac{4}{3}\pi r^3$

$\frac{dv}{dt}=4\pi r^2\frac{dr}{dt}$

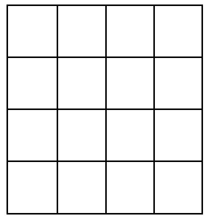
$81=4\pi r^2\times\frac{1}{4\pi}$

$r^2=81$

$r=9$

surface area of chocolate $=4\pi(r-1)^2=256\pi$

10. A board has 16 squares as shown in the figure :



Out of these 16 squares, two squares are chosen at random. The probability that they have no side in common is :

(1) $\frac{4}{5}$ (2) $\frac{7}{10}$

(3) $\frac{3}{5}$ (4) $\frac{23}{30}$

Ans. (1)

Sol. Total ways for selecting any two squares $={}^{16}C_2$

$=120$

Total ways for selecting common side squares

$=\underbrace{3\times4}_{\text{Horizontal side}}+\underbrace{3\times4}_{\text{vertical side}}$

$=24$

so required probability

$=1-\frac{24}{120}$

$=\frac{4}{5}$

11. Let x = x(y) be the solution of the differential equation

$y=\left(x-y\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right),\ y>0$ and $x(1)=\frac{\pi}{2}$.

Then cos(x(2)) is equal to :

(1) $1-2(\log_e 2)^2$ (2) $2(\log_e 2)^2-1$

(3) $2(\log_e 2)-1$ (4) $1-2(\log_e 2)$

Ans. (2)

Sol. $ydy=(xdy-ydx)\sin\left(\frac{x}{y}\right)$

$\frac{dy}{y}=\left(\frac{xdy-ydx}{y^2}\right)\sin\left(\frac{x}{y}\right)$

$\frac{dy}{y}=\sin\left(\frac{x}{y}\right)d\left(-\frac{x}{y}\right)$

$\Rightarrow\ \ell ny=\cos\frac{x}{y}+C$

$x(1) = \frac{\pi}{2} \Rightarrow 0 = \cos\frac{\pi}{2} + C \Rightarrow C=0$

$\ell ny = \cos\frac{x}{y}$

but $y = 2 \Rightarrow \cos\frac{x}{2} = \ell n2$

$\cos x = 2\cos^2\frac{x}{2} - 1$

$= 2(\ell n2)^2 - 1$

12. Let the range of the function

$f(x) = 6 + 16 \cos x \cdot \cos\left(\frac{\pi}{3} - x\right) \cdot \cos\left(\frac{\pi}{3} + x\right) \cdot$

$\sin 3x \cdot \cos 6x$, $x \in R$ be $[\alpha, \beta]$. Then the distance of the point (α, β) from the line $3x + 4y + 12 = 0$ is :

(1) 11 (2) 8

(3) 10 (4) 9

Ans. **(1)**

Sol. $f(x) = 6 + 16\left(\frac{1}{4}\cos 3x\right)\sin 3x . \cos 6x$

$= 6 + 4 \cos 3x \sin 3x \cos 6x$

$= 6 + \sin 12x$

Range of f(x) is [5, 7]

$(\alpha, \beta) \equiv (5, 7)$

distance $= \left|\frac{15+28+12}{5}\right| = 11$

13. Let the shortest distance from (a, 0), a > 0, to the parabola $y^2 = 4x$ be 4. Then the equation of the circle passing through the point (a, 0) and the focus of the parabola, and having its centre on the axis of the parabola is:

(1) $x^2 + y^2 - 6x + 5 = 0$

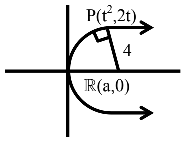
(2) $x^2 + y^2 - 4x + 3 = 0$

(3) $x^2 + y^2 - 10x + 9 = 0$

(4) $x^2 + y^2 - 8x + 7 = 0$

Ans. **(1)**

Sol. Normal at P



$y + tx = 2t + t^3$

↑

(a, 0)

$at = 2t + t^3$

$a = 2 + t^2$

$\mathbb{R}\ (2 + t^2, 0)$

$P\mathbb{R} = 4 \Rightarrow 4 + 4t^2 = 16$

$4t^2 = 12 \Rightarrow t^2 = 3$

$a = 5 \quad \mathbb{R}\ (5, 0)$

Focus (1, 0)

(1, 0) & (5, 0) will be tha end pts. of diameter

$\Rightarrow$ Eg^n of circle is

$(x - 1)(x - 5) + y^2 = 0$

$x^2 + y^2 - 6x + 5 = 0$

14. Let $X = R \times R$. Define a relation R on X as:

$(a_1, b_1)\ R\ (a_2, b_2) \Leftrightarrow b_1 = b_2$.

Statement-I: R is an equivalence relation.

Statement-II: For some $(a, b) \in X$, the set $S = \{(x, y) \in X : (x, y)\ R\ (a, b)\}$ represents a line parallel to $y = x$.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **Statement-I** and **Statement-II** are false.

(2) **Statement-I** is true but **Statement-II** is false.

(3) Both **Statement-I** and **Statement-II** are true.

(4) **Statement-I** is false but **Statement-II** is true.

Ans. **(2)**

Sol. **Statement – I :**

Reflexive : $(a_1,b)\ R(a_1, b_1) \Rightarrow b_1 = b_1$ True

Symmetric : $(a_1,b_1)\ R\ (a_2, b_2) \Rightarrow b_1 = b_2$, $(a_2,b_2)\ R\ (a_1, b_1) \Rightarrow b_2 = b_1$ } True

Transitive : $(a_1, b_1)\ R\ (a_2, b_2) \Rightarrow b_1 = b_2$ & $(a_2, b_2)\ R\ (a_3, b_3)\ b_2 = b_3$ } $b_1 = b_3$ $\Rightarrow (a_1, b_1)\ R\ (a_3. b_3) \Rightarrow$ True

Hence Relation R is an equivence relation Statement-I is true.

For statement – II $\Rightarrow y = b$ so False

15. The length of the chord of the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$, whose mid-point is $\left(1,\frac{1}{2}\right)$, is:

(1) $\frac{2}{3}\sqrt{15}$ (2) $\frac{5}{3}\sqrt{15}$

(3) $\frac{1}{3}\sqrt{15}$ (4) $\sqrt{15}$

Ans. **(1)**

Sol. $T = S_1$

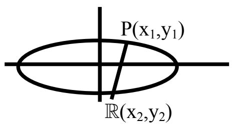
$$\frac{x.1}{4}+\frac{y.\frac{1}{2}}{2}=\frac{1}{4}+\frac{1}{8}$$

$$x + y = \frac{3}{2}$$

solve with ellipse

$$P_{\mathbb{R}} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{2}\,|x_2-x_1|$$



$$y_2 = \frac{3}{2} - x_2$$

$$y_1 = \frac{3}{2} - x_1$$

$$y_2 - y_1 = x_2 - x_1$$

$$x^2 + 2y^2 = 4$$

$$x^2 + 2\left(\frac{3}{2}-x\right)^2 = 4$$

$$6x^2 - 12x + 1 = 0$$

$$x_1 + x_2 = 2$$

$$x_1x_2 = 1/6$$

$$|x_2-x_1| = \sqrt{(x_2+x_1)^2-4x_1x_2}$$

$$= \sqrt{4-4/6}$$

$$PR = \sqrt{2}.2.\frac{\sqrt{5}}{\sqrt{2}\sqrt{3}} = \frac{2}{3}\sqrt{15}$$

$$= 2\sqrt{5/6}$$

16. Let $A = [a_{ij}]$ be a 3×3 matrix such that $A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}$ and $A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$, then a_{23} equals:

(1) –1 (2) 0

(3) 2 (4) 1

Ans. **(1)**

Sol. Let $A = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$

$$A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix} \Rightarrow \begin{bmatrix}a_{12}\\a_{22}\\a_{32}\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix} \Rightarrow \begin{matrix}a_{22}=0; a_{12}=0\\ a_{32}=1\end{matrix}$$

$$A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix} \Rightarrow \begin{matrix}4a_{11}+a_{12}+3a_{13}=0\\ 4a_{21}+a_{22}+3a_{23}=1 \Rightarrow 4a_{21}+3a_{23}=1\\ 4a_{31}+a_{32}+3a_{33}=0\end{matrix}$$

$$A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix} \Rightarrow \begin{matrix}2a_{11}+a_{12}+2a_{13}=1\\ 2a_{21}+a_{22}+2a_{23}=0 \Rightarrow a_{21}+a_{23}=0\\ 2a_{31}+a_{32}+2a_{33}=0\end{matrix}$$

$$-4a_{23} + 3a_{23} = 1 \Rightarrow a_{23} = -1$$

17. The number of complex numbers z, satisfying $|z| = 1$ and $\left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right|=1$, is :

(1) 6 (2) 4

(3) 10 (4) 8

Ans. **(4)**

Sol. $z = e^{i\theta}$

$$\frac{z}{\bar{z}} = e^{i2\theta}$$

$$\left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right|=1 \Rightarrow \left|e^{i2\theta}+e^{-i2\theta}\right|=1 \Rightarrow |\cos 2\theta| = \frac{1}{2}$$

8 solution

18. If the square of the shortest distance between the lines $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z+3}{-3}$ and $\frac{x+1}{2}=\frac{y+3}{4}=\frac{z+5}{-5}$ is $\frac{m}{n}$, where m, n are coprime numbers, then m + n is equal to:

(1) 6 (2) 9

(3) 21 (4) 14

Ans. **(2)**

Sol. $\vec{a} = (2,1,-3)$

$\vec{b} = (-1,-3,-5)$

$$\vec{p}\times\vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$$

$= 2\hat{i} - \hat{j}$

$\vec{b} - \vec{a} = -3\hat{i} - 4\hat{j} - 2\hat{k}$

$$S_d = \frac{\left|(\vec{b}-\vec{a}).(\vec{p}\times\vec{q})\right|}{|\vec{p}\times\vec{q}|}$$

$$= \frac{2}{\sqrt{5}}$$

$$(S_d)^2 = \frac{4}{5}$$

$m = 4, n = 5 \Rightarrow m + n = 9$

19. If $I = \int_0^{\frac{\pi}{2}} \frac{\sin^{\frac{3}{2}} x}{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x} dx$,

then $\int_0^{21} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ equals:

(1) $\frac{\pi^2}{16}$ (2) $\frac{\pi^2}{4}$

(3) $\frac{\pi^2}{8}$ (4) $\frac{\pi^2}{12}$

Ans. **(1)**

Sol. For I

Apply king (P– 5) and add

$$2I = \int_0^{\pi/2} dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$

$$I_2 = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

Apply king and add

$$I_2 = \frac{\pi}{4}\int_0^{\pi/2} \frac{\tan x \sec^2 x dx}{\tan^4 x + 1}$$

put $\tan^2 x = t$

$$\frac{\pi}{8}\int_0^{\infty} \frac{dt}{t^2+1}$$

$$= \frac{\pi}{8}.\frac{\pi}{2} = \frac{\pi^2}{16}$$

20. $\lim\limits_{x\to\infty} \frac{(2x^2-3x+5)(3x-1)^{\frac{x}{2}}}{(3x^2+5x+4)\sqrt{(3x+2)^x}}$ is equal to:

(1) $\frac{2}{\sqrt{3e}}$ (2) $\frac{2e}{\sqrt{3}}$

(3) $\frac{2e}{3}$ (4) $\frac{2}{3\sqrt{e}}$

Ans. **(4)**

Sol. $$\lim_{x\to\infty} \frac{\left(2-\frac{3}{x}+\frac{5}{x^2}\right)\left(1-\frac{1}{3x}\right)^{x/2}}{\left(3+\frac{5}{x}+\frac{4}{x^2}\right)\left(1+\frac{2}{3x}\right)^{x/2}}$$

$$= \lim_{x\to\infty} \frac{2}{3}.\frac{e^{\frac{x}{2}\left(1-\frac{1}{3x}-1\right)}}{e^{\frac{x}{2}\left(1+\frac{2}{3x}-1\right)}}$$

$$= \frac{2}{3}.\frac{e^{-\frac{1}{6}}}{e^{1/3}} = \frac{2}{3}e^{-\frac{1}{2}}$$

SECTION-B

21. The number of ways, 5 boys and 4 girls can sit in a row so that either all the boys sit together or no two boys sit together, is ________.

Ans. **(17280)**

Sol. A : number of ways that all boys sit together = 5! × 5!

B : number of ways if no 2 boys sit together = 4! × 5!

$A \cap B = \phi$

Required no. of ways = 5! × 5! + 4! × 5! = 17280

22. Let α, β be the roots of the equation $x^2 - ax - b = 0$ with Im(α) < Im(β). Let $P_n = \alpha^n - \beta^n$. If $P_3 = -5\sqrt{7}i$, $P_4 = -3\sqrt{7}i$, $P_5 = 11\sqrt{7}i$ and $P_6 = 45\sqrt{7}i$, then $|\alpha^4 + \beta^4|$ is equal to ________.

Ans. **(31)**

Sol. $\alpha + \beta = a \quad \alpha\beta = -b$

$P_6 = aP_5 + bP_4$

$45\sqrt{7}i = a \times 11\sqrt{7}i + b\left(-3\sqrt{7}\right)i$

$45 = 11a - 3b$ ...(1)

and $P_5 = aP_4 + bP_3$

$$11\sqrt{7}i = a\left(-3\sqrt{7}i\right) + b\left(-5\sqrt{7}i\right)$$

$11 = -3a - 5b$ ...(2)

$a = 3, b = -4$

$$\left|\alpha^4 + \beta^4\right| = \sqrt{\left(\alpha^4 - \beta^4\right)^2 + 4\alpha^4\beta^4}$$

$$= \sqrt{-63 + 4.4^4}$$

$$= \sqrt{-63 + 1024} = \sqrt{961} = 31$$

23. The focus of the parabola $y^2 = 4x + 16$ is the centre of the circle C of radius 5. If the values of λ, for which C passes through the point of intersection of the lines $3x - y = 0$ and $x + \lambda y = 4$, are λ_1 and λ_2, $\lambda_1 < \lambda_2$, then $12\lambda_1 + 29\lambda_2$ is equal to _______.

Ans. **(15)**

Sol. $y^2 = 4(x + 4)$

Equation of circle

$(x + 3)^2 + y^2 = 25$

Passes through the point of intersection of two lines $3x - y = 0$ and $x + \lambda y = 4$

$\left(\dfrac{4}{3\lambda+1}, \dfrac{12}{3\lambda+1}\right)$, we get

$$\lambda = -\frac{7}{6}, 1$$

$12\lambda_1 + 29\lambda_2$

$-14 + 29 = 15$

24. The variance of the numbers 8, 21, 34, 47, …, 320, is _________.

Ans. **(8788)**

Sol. $8 + (n - 1)13 = 320$

$13n = 325$

$n = 25$

no. of terms = 25

$$\text{mean} = \frac{\Sigma x_i}{n} = \frac{8 + 21 + ... + 320}{25} = \frac{\frac{25}{2}(8 + 320)}{25}$$

$$\text{variance } \sigma^2 = \frac{\Sigma x_i^2}{n} - (\text{mean})^2$$

$$= \frac{8^2 + 21^2 + ... + 320^2}{13} - (164)^2$$

$= 8788$

25. The roots of the quadratic equation $3x^2 - px + q = 0$ are 10^{th} and 11^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88, then $q - 2q$ is equal to _________.

Ans. **(474)**

Sol. $S_{11} = \dfrac{11}{2}(2a + 10d) = 88$

$a + 5d = 8$

$$a = 8 - 5 \times \frac{3}{2} = \frac{1}{2}$$

Roots are

$$T_{10} = a + 9d = \frac{1}{2} + 9 \times \frac{3}{2} = 14$$

$$T_{11} = a + 10d = \frac{1}{2} + 10 \times \frac{3}{2} = \frac{31}{2}$$

$$\frac{p}{3} = T_{10} + T_{11} = 14 + \frac{31}{2} = \frac{59}{2}$$

$$p = \frac{177}{2}$$

$$\frac{q}{3} = T_{10} \times T_{11} = 7 \times 31 = 217$$

$q = 651$

$q - 2p$

$= 651 - 177$

$= 474$