JEE–MAIN EXAMINATION – APRIL 2025

(HELD ON WEDNESDAY 2nd APRIL 2025) **TIME : 3:00 PM TO 6:00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. If the image of the point P(1, 0, 3) in the line joining the points A(4, 7, 1) and B(3, 5, 3) is Q(α, β, γ), then α + β + γ is equal to

(1) $\frac{47}{3}$ (2) $\frac{46}{3}$

(3) 18 (4) 13

Ans. (2)

Sol. P(1, 0, 3)

A(4, 7, 1), B(3, 5, 3)

Line AB $\Rightarrow \frac{x-3}{1}=\frac{y-5}{2}=\frac{z-3}{-2}=\lambda$

Let foot of perpendicular of P on AB be

R ≡ (λ + 3, 2λ + 5, –2λ + 3)

⇒ (λ + 3 – 1) (1) + (2λ + 5 – 0) (2) + (–2λ + 3 – 3) (–2) = 0

⇒ λ + 2 + 4λ + 10 + 4λ = 0

$\Rightarrow \lambda=-\frac{4}{3}$

$\Rightarrow R\equiv\left(\frac{5}{3},\frac{7}{3},\frac{17}{3}\right)$

$Q\equiv\left(\frac{10}{3}-1,\frac{14}{3}-0,\frac{34}{3}-3\right)\equiv\left(\frac{7}{3},\frac{14}{3},\frac{25}{3}\right)$

$\Rightarrow \alpha+\beta+\gamma=\frac{7+14+25}{3}=\frac{46}{3}$

2. Let $f:[1,\infty)\to[2,\infty)$ be a differentiable function, If $10\int_1^x f(t)dt=5xf(x)-x^5-9$ for all x ≥ 1, then the value of $f(3)$ is :

(1) 18 (2) 32

(3) 22 (4) 26

Ans. (2)

Sol. $10\frac{d}{dx}\int_1^x f(t)dt=\frac{d}{dx}\left(5xf(x)-x^5-9\right)$

⇒ 10f(x) = 5f(x) + 5x f ′ (x) – 5x^4

⇒ f(x) + x^4 = x f ′ (x)

$\Rightarrow y+x^4=x\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}+y\left(-\frac{1}{x}\right)=x^3$

$\Rightarrow ye^{\int-\frac{1}{x}dx}=\int x^3e^{\int-\frac{1}{x}dx}+c$

$\Rightarrow ye^{-\ell n|x|}=\int x^3e^{-\ell n|x|}+c$

$\Rightarrow \frac{y}{|x|}=\int\frac{x^3}{|x|}+c$

$\Rightarrow \frac{y}{x}=\int x^2+c$

$\Rightarrow \frac{y}{x}=\frac{x^3}{3}+c$

Put x = 1 in given equation

⇒ 0 = 5f(1) – 1 – 9 ⇒ f(1) = 2

$\Rightarrow \frac{2}{1}=\frac{1}{3}+c\Rightarrow c=\frac{5}{3}$

$\Rightarrow \frac{f(3)}{3}=\frac{27}{3}+\frac{5}{3}$

⇒ f(3) = 32

3. The number of terms of an A.P. is even; the sum of all the odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is :

(1) 4 (2) 10

(3) 6 (4) 8

Ans. (1)

Sol. $a_2 + a_4 + ... + a_n = 30$ ....(1)

$a_1 + a_3 + ... + a_{n-1} = 24$ ....(2)

(1) – (2)

$(a_2 - a_1) + (a_4 - a_3) ... (a_n - a_{n-1}) = 6$

$\Rightarrow \frac{n}{2}d = 6 \Rightarrow nd = 12$

$a_n - a_1 = (n-1)d = \frac{21}{2}$

$\Rightarrow nd - d = \frac{21}{2} \Rightarrow 12 - \frac{21}{2} = d$

$\Rightarrow d = \frac{3}{2}$, $n = 8$

Sum of odd terms $= \frac{4}{2}\left[2a + (4-1)3\right] = 24$

$\Rightarrow a = \frac{3}{2}$

A.P. $\Rightarrow \frac{3}{2}, 3, \frac{9}{2}, 6, \frac{15}{2}, 9, \frac{21}{2}, 12$

no. of integer terms = 4

4. Let A = {1, 2, 3,...., 10} and R be a relation on A such that R = {(a, b) : a = 2b + 1}. Let (a_1, a_2), (a_2, a_3), (a_3, a_4), ..., (a_k, a_{k+1}) be a sequence of k elements of R such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer k, for which such a sequence exists, is equal to :

(1) 6 (2) 7

(3) 5 (4) 8

Ans. **(3)**

Sol. $a = 2b + 1$

$2b = a - 1$

$R = \{(3, 1), (5, 2), ..., (99, 49)\}$

Let $(2m + 1, m)$, $(2\lambda - 1, \lambda)$ are such ordered pairs.

According to the condition

$m = 2\lambda - 1 \Rightarrow m$ = odd number

$\Rightarrow 1^{st}$ element of ordered pair (a, b)

$a = 2(2\lambda - 1) + 1 = 4\lambda - 1$

Hence $a \in \{3, 7, ..., 99\}$

$\Rightarrow \lambda \in \{1, 2, ..., 25\}$

$\Rightarrow$ set of sequence

$\left\{(4\lambda - 1, 2\lambda - 1), (2\lambda - 1, \lambda - 1), \left(\lambda - 1, \frac{\lambda - 2}{2}\right), ...\right\}$

2^{nd} element of each ordered pair $= \frac{\lambda - 2^{r-2}}{2^{r-2}}$

For maximum number of ordered pairs in such sequence

$\frac{\lambda - 2^{r-2}}{2^{r-2}} = 1$ or 2 ; $1 \le \lambda \le 25$

$\lambda = 2^{r-1}$ or $\lambda = 3.2^{r-2}$

Case-I : $\lambda = 2r - 1$

$\lambda = 2, 2^2, 2^3, 2^4$

$r = 2, 3, 4, 5$

Hence maximum value of r is 5 when $\lambda = 16$

Case-II : $\lambda = 3.2^{r-2}$

$\lambda = 3, 6, 12, 24$

$r = 2, 3, 4, 5$

Hence maximum value of r is 5 when $\lambda = 24$

5. If the length of the minor axis of an ellipse is equal to one fourth of the distance between the foci, then the eccentricity of the ellipse is :

(1) $\frac{4}{\sqrt{17}}$ (2) $\frac{\sqrt{3}}{16}$

(3) $\frac{3}{\sqrt{19}}$ (4) $\frac{\sqrt{5}}{7}$

Ans. **(1)**

Sol. $2b = \frac{1}{4}(2ae)$

$\frac{b}{a} = \frac{e}{4}$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$e = \sqrt{1 - \frac{e^2}{16}}$

$e^2\left(1 + \frac{1}{16}\right) = 1$

$e = \frac{4}{\sqrt{17}}$

6. The line L_1 is parallel to the vector $\vec{a}=-3\hat{i}+2\hat{j}+4\hat{k}$ and passes through the point (7, 6, 2) and the line L_2 is parallel to the vector $\vec{b}=2\hat{i}+\hat{j}+3\hat{k}$ and passes through the point (5, 3, 4). The shortest distance between the lines L_1 and L_2 is :

(1) $\frac{23}{\sqrt{38}}$ (2) $\frac{21}{\sqrt{57}}$

(3) $\frac{23}{\sqrt{57}}$ (4) $\frac{21}{\sqrt{38}}$

Ans. (1)

Sol. $L_1:(7\hat{i}+6\hat{j}+2\hat{k})+\lambda(-3\hat{i}+2\hat{j}+4\hat{k})$

$L_2:(5\hat{i}+3\hat{j}+4\hat{k})+\lambda(2\hat{i}+\hat{j}+3\hat{k})$

Distance between skew lines

$$=\frac{(2\hat{i}+3\hat{j}-2\hat{k}).(2\hat{i}+17\hat{j}-7\hat{k})}{\sqrt{342}}$$

$$=\frac{69}{\sqrt{342}}=\frac{69}{3\sqrt{38}}=\frac{23}{\sqrt{38}}$$

7. Let (a, b) be the point of intersection of the curve $x^2=2y$ and the straight line $y-2x-6=0$ in the second quadrant. Then the integral $I=\int_a^b \frac{9x^2}{1+5^x}dx$ is equal to :

(1) 24 (2) 27

(3) 18 (4) 21

Ans. (1)

Sol. $x^2 = 2y$ & $y = 2x + 6$

$x^2 = 4x + 12$

$x^2-4x-12=0 \Rightarrow \begin{array}{c|c} x=6 & \text{if } x=-2 \\ y=18 & y=2 \end{array}$

$\therefore$ (6, 18) & (–2, 2)

Here (6, 18) Rejected because (a, b) lies in 2^{nd} quadrant

$\therefore a = -2$ & $b = 2$

$$\therefore I=\int_{-2}^{2}\frac{9x^2}{1+5^x}dx=\int_{-2}^{2}\frac{9.5^x.x^2}{1+5^x}dx$$

$$\therefore 2I=\int_{-2}^{2}9x^2dx=18\int_0^2 x^2dx=18\left(\frac{x^3}{3}\right)_0^2$$

$2I = 48$

$\therefore \boxed{I=24}$

8. If the system of equation

$2x + \lambda y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \mu z = 9$

has infinitely many solutions, then $(\lambda^2+\mu^2)$ is equal to :

(1) 22 (2) 18

(3) 26 (4) 30

Ans. (3)

Sol. $\Delta=0\Rightarrow\begin{vmatrix}2&\lambda&3\\3&2&-1\\4&5&\mu\end{vmatrix}=0$

$\Rightarrow 2(2\mu+5)+\lambda(-4-3\mu)+3(7)=0$

$\Rightarrow 4\mu-3\lambda\mu-4\lambda+31=0$ ......(1)

$\Delta_3=0\Rightarrow\begin{vmatrix}2&\lambda&5\\3&2&7\\4&5&9\end{vmatrix}=0$

$\Rightarrow 2(-17)+\lambda(1)+5(7)=0$

$\Rightarrow \lambda=-1$

from equation (1)

$4\mu+3\mu+4+31=0\Rightarrow\boxed{\mu=-5}$

$\therefore \boxed{\lambda^2+\mu^2=26}$

9. If $\theta\in\left[-\frac{7\pi}{6},\frac{4\pi}{3}\right]$, then the number of solutions of $\sqrt{3}\,\text{cosec}^2\theta-2(\sqrt{3}-1)\text{cosec}\theta-4=0$, is equal to

(1) 6 (2) 8

(3) 10 (4) 7

Ans. (1)

Sol. $$\text{cosec}\theta=\frac{2(\sqrt{3}-1)\pm\sqrt{4(3+1-2\sqrt{3})+16\sqrt{3}}}{2\sqrt{3}}$$

$$=\frac{2(\sqrt{3}-1)\pm\sqrt{16+8\sqrt{3}}}{2\sqrt{3}}$$

$$=\frac{2(\sqrt{3}-1)\pm(2+2\sqrt{3})}{2\sqrt{3}}$$

$\operatorname{cosec}\theta = 2$ or $\frac{-2}{\sqrt{3}}$

$\therefore \sin\theta = \frac{1}{2}$ or $\frac{-\sqrt{3}}{2}$

$\therefore \sin\theta = \frac{1}{2}$ has 3 solutions & also $\sin\theta = \frac{-\sqrt{3}}{2}$

has 3 solutions in $\left[\frac{-7\pi}{6}, \frac{4\pi}{3}\right]$

10. Given three indentical bags each containing 10 balls, whose colours are as follows :

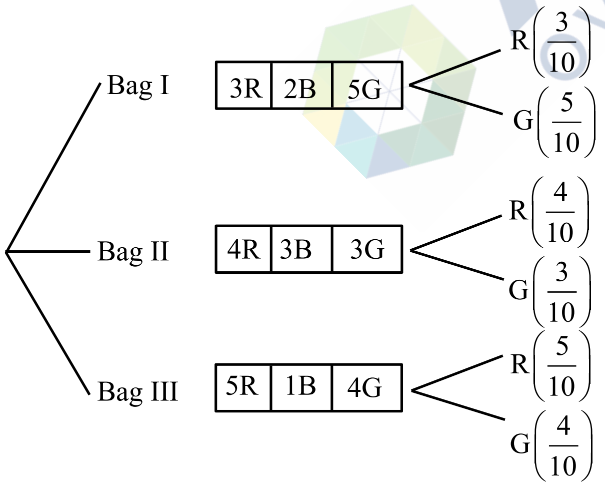
	Red	Blue	Green
Bag I	3	2	5
Bag II	4	3	3
Bag III	5	1	4

A person chooses a bag at random and takes out a ball. If the ball is Red, the probability that it is from bag I is p and if the balls is Green, the probability that it is from bag III is q, then the value of $\left(\frac{1}{p}+\frac{1}{q}\right)$ is :

(1) 6
(2) 9
(3) 7
(4) 8

Ans. (3)

Sol.



$$p = P\left(\frac{B_I}{R}\right) = \frac{\frac{1}{3}\left(\frac{3}{10}\right)}{\frac{1}{3}\left(\frac{3}{10}+\frac{4}{10}+\frac{5}{10}\right)} = \frac{1}{4}$$

$$q = \left(\frac{B_{III}}{G}\right) = \frac{\frac{1}{3}\left(\frac{4}{10}\right)}{\frac{1}{3}\left(\frac{5}{10}+\frac{3}{10}+\frac{4}{10}\right)} = \frac{1}{3}$$

$$\therefore \frac{1}{p}+\frac{1}{q} = 7$$

11. If the mean and the variance of 6, 4, a, 8, b, 12, 10, 13 are 9 and 9.25 respectively, then a + b + ab is equal to :

(1) 105
(2) 103
(3) 100
(4) 106

Ans. (2)

Sol. $\because$ mean = 9

$\therefore 53 + a + b = 72$

$\Rightarrow a + b = 19$

$\because \sigma^2 = \frac{37}{4}$ and $(\bar{X})^2 + \sigma^2 = \frac{\sum x_i^2}{N}$

$\Rightarrow 81 + \frac{37}{4} = \frac{529 + a^2 + b^2}{8}$

$\Rightarrow 648 + 74 = 529 + a^2 + b^2$

$\Rightarrow a^2 + b^2 = 193$

$\because a + b = 19 \Rightarrow a^2 + b^2 + 2ab = 361$

$\Rightarrow 2ab = 168$

$\Rightarrow ab = 84$

$\therefore a + b + ab = 103$

12. If the domain of the function $f(x) = \frac{1}{\sqrt{10+3x-x^2}} + \frac{1}{\sqrt{x+|x|}}$ is (a, b), then $(1 + a)^2 + b^2$ is equal to :

(1) 26
(2) 29
(3) 25
(4) 30

Ans. (1)

Sol. $x + |x| > 0 \quad \Rightarrow x \in (0, \infty)$ ....(1)

$\& \ 10 + 3x - x^2 > 0$

$\Rightarrow x^2 - 3x - 10 < 0$

$\Rightarrow x \in (-2, 5)$ ....(2)

from (1) & (2) $\quad x \in (0, 5)$

$\therefore a = 0 \ \& \ b = 5$

$\therefore (1 + a^2) + b^2 = 1 + 25 = 26$

13. $4\int_0^1\left(\frac{1}{\sqrt{3+x^2}+\sqrt{1+x^2}}\right)dx - 3\log_e\left(\sqrt{3}\right)$ is equal to :

(1) $2+\sqrt{2}+\log_e\left(1+\sqrt{2}\right)$

(2) $2-\sqrt{2}-\log_e\left(1+\sqrt{2}\right)$

(3) $2+\sqrt{2}-\log_e\left(1+\sqrt{2}\right)$

(4) $2-\sqrt{2}+\log_e\left(1+\sqrt{2}\right)$

Ans. (2)

Sol. $4\int_0^1\frac{1}{\sqrt{3+x^2}+\sqrt{1+x^2}}dx - 3\ln\sqrt{3}$

$= 4\int_0^1\frac{\sqrt{3+x^2}-\sqrt{1+x^2}}{(3+x^2)-(1-x^2)}dx - \frac{3}{2}\ln 3$

$= 2\left[\left\{\frac{x}{2}\sqrt{3+x^2}+\frac{3}{2}\ln\left(x+\sqrt{3+x^2}\right)\right\}_0^1\right.$

$\left.-\left\{\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\ln(x+\sqrt{1+x^2})\right\}_0^1\right] - \frac{3}{2}\ln 3$

$= 2\left[\left\{\frac{1}{2}\sqrt{4}+\frac{3}{2}\ln(1+\sqrt{4})\right\}-\left\{0+\frac{3}{2}\ln\sqrt{3}\right\}\right.$

$\left.-\left\{\frac{1}{2}\sqrt{2}+\frac{1}{2}\ln(1+\sqrt{2})\right\}+\left\{0+\frac{1}{2}(0)\right\}\right]-\frac{3}{2}\ln 3$

$=2\left[1+\frac{3}{2}\ln 3-\frac{3}{4}\ln 3-\frac{1}{\sqrt{2}}-\frac{1}{2}\ln(1+\sqrt{2})\right]-\frac{3}{2}\ln 3$

$=2+3\ln 3-\frac{3}{2}\ln 3-\sqrt{2}-\ln(1+\sqrt{2})-\frac{3}{2}\ln 3$

$=2-\sqrt{2}-\ln(1+\sqrt{2})$

14. If $\lim_{x\to 0}\frac{\cos(2x)+a\cos(4x)-b}{x^4}$ is finite, then (a+b) is equal to :

(1) $\frac{1}{2}$ (2) 0

(3) $\frac{3}{4}$ (4) –1

Ans. (1)

Sol. $\lim_{x\to 0}\frac{\cos 2x+a\cos 4x-b}{x^4}$ = finite

$$L=\frac{\left\{1-\frac{(2x)^2}{\lfloor 2}+\frac{(2x)^4}{\lfloor 4}....\right\}+a\left\{1-\frac{(4x)^2}{\lfloor 2}+\frac{(4x)^4}{\lfloor 4}....\right\}-b}{x^4}$$

$$L=\frac{(1+a-b)-x^2(2+8a)+x^4\left(\frac{2}{3}+\frac{32}{3}a\right)+x^6(\,)....}{x^4}$$

$\therefore$ $1+a-b=0$ and $2+8a=0$ $\Rightarrow a=-\frac{1}{4}$

$b=a+1$

$=-\frac{1}{4}+1=\frac{3}{4}$

$\therefore$ $a+b=-\frac{1}{4}+\frac{3}{4}=\frac{1}{2}$

15. If $\sum_{r=0}^{10}\left(\frac{10^{r+1}-1}{10^r}\right).{}^{11}C_{r+1}=\frac{\alpha^{11}-11^{11}}{10^{10}}$, then α is equal to :

(1) 15 (2) 11

(3) 24 (4) 20

Ans. (4)

Sol. $\sum_{r=0}^{10}\left(\frac{10^{r-1}-1}{10^r}\right){}^{11}C_{r+1}$

$=\sum_{r=0}^{10}\left(10-\frac{1}{10^r}\right){}^{11}C_{r+1}$

$=10\sum_{r=0}^{10}{}^{11}C_{r+1}-10\sum\left({}^{11}C_{r+1}\left(\frac{1}{10}\right)^{r+1}\right)$

$=10\left[{}^{11}C_1+{}^{11}C_2+.....+{}^{11}C_{11}\right]$

$-10\left[{}^{11}C_1\left(\frac{1}{10}\right)^1+{}^{11}C_2\left(\frac{1}{10}\right)^2+.....+{}^{11}C_{11}\left(\frac{1}{10}\right)^{11}\right]$

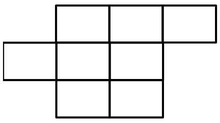
$=10\left[2^{11}-1\right]-10\left[\left(1+\frac{1}{10}\right)^{11}-1\right]$

$=10(2)^{11}-10-\frac{11^{11}}{10^{10}}+10$

$=\frac{(20)^{11}-11^{11}}{10^{10}}$

$\therefore$ $\alpha=20$

16. The number of ways, in which the letters A, B, C, D, E can be placed in the 8 boxes of the figure below so that no row remains empty and at most one letter can be placed in a box, is :

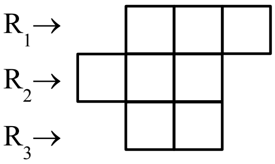


(1) 5880 (2) 960

(3) 840 (4) 5760

Ans. **(4)**

Sol.



= Total – [(All in R_1 and R_3) + (All in R_2 and R_3) + (All in R_1 and R_2)]

$= {}^8C_5 \cdot \lfloor\underline{5} - \{\lfloor\underline{5} + \lfloor\underline{5} + {}^6C_5 \cdot \lfloor\underline{5}\}$

$= \lfloor\underline{5}(56 - 1 - 1 - 6) = 120(48)$

= 5760

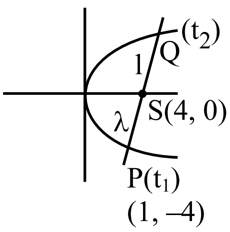
17. Let the point P of the focal chord PQ of the parabola $y^2 = 16x$ be $(1, -4)$. If the focus of the parabola divides the chord PQ in the ratio m : n, gcd(m, n) = 1, then $m^2 + n^2$ is equal to :

(1) 17 (2) 10

(3) 37 (4) 26

Ans. **(1)**

Sol. $y^2 = 16x$; a = 4 focus $S \equiv (4, 0)$



$2at_1 = -4$

$\Rightarrow 2(4)t_1 = -4$

$\Rightarrow t_1 = -\frac{1}{2}$

$\because t_1t_2 = -1$

$\Rightarrow t_2 = 2$

$\therefore Q\left(at_2^2, 2at_2\right) = (16, 16)$

Let, S divides PQ internally in λ : 1 ratio

$\therefore \frac{16\lambda - 4}{\lambda + 1} = 0$

$\lambda = \frac{1}{4} = \frac{m}{n}$

$\therefore m^2 + n^2 = 1 + 16 = 17$

18. Let $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$ and a vector $\vec{c}$ be such that $\left(\vec{a} - \vec{c}\right) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12\hat{k}$ and $\vec{a}.\vec{c} = 3$. If $\vec{b} \times \vec{c} = \vec{d}$, then $\left|\vec{a}.\vec{d}\right|$ is equal to :

(1) 18 (2) 12

(3) 9 (4) 15

Ans. **(4)**

Sol. $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$

$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 3 & 2 & 5 \end{vmatrix}$

$= -17\hat{i} - 7\hat{j} + 13\hat{k}$

$(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3j + 12\hat{k}$

$\Rightarrow (\vec{a} \times \vec{b}) - (\vec{c} \times \vec{b}) = -18\hat{i} - 3\hat{j} + 12\hat{k}$

$\Rightarrow \vec{b} \times \vec{c} = (-18\hat{i} - 3\hat{j} + 12\hat{k}) - (\vec{a} \times \vec{b})$

$= (-18\hat{i} - 3\hat{j} + 12\hat{k}) - (-17i - 7\hat{j} + 13\hat{k})$

$\vec{b} \times \vec{c} = -\hat{i} + 4\hat{j} - \hat{k}$

$\therefore \vec{a} \cdot \vec{d} = \vec{a} \cdot (\vec{b} \times \vec{c}) = (2i - 3j + k) \cdot (-\hat{i} + 4\hat{j} - \hat{k})$

= –2 – 12 – 1 = –15

$\therefore \left|\vec{a} \cdot \vec{d}\right| = 15$

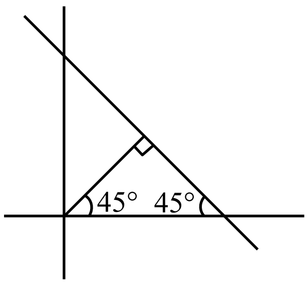
19. Let the area of the triangle formed by a straight Line L : x + by + c = 0 with co-ordinate axes be 48 square units. If the perpendicular drawn from the origin to the line L makes an angle of 45° with the positive x-axis, then the value of $b^2 + c^2$ is:

(1) 90 (2) 93

(3) 97 (4) 83

Ans. **(3)**

Sol. $\frac{x}{-c}+\frac{y}{-c/b}=1$



$\therefore$ area of triangle $=\frac{1}{2}\left|\frac{c^2}{b}\right|=48$

$\left|\frac{c^2}{b}\right|=96$

$\because -c=-\frac{c}{b}$

$\Rightarrow b=1 \quad \therefore c^2=96$

$\therefore b^2+c^2=97$

20. Let A be a 3 × 3 real matrix such that $A^2(A-2I)-4(A-I)=O$, where I and O are the identity and null matrices, respectively. If $A^5=\alpha A^2+\beta A+\gamma I$, where α, β and γ are real constants, then $\alpha+\beta+\gamma$ is equal to:

(1) 12 (2) 20
(3) 76 (4) 4

Ans. (1)

Sol. $A^3-2A^2-4A+4I=0$

$A^3=2A^2+4A-4I$

$A^4=2A^3+4A^2-4A$

$=2(2A^2+4A-4I)+4A^2-4A$

$A^4=8A^2+4A-8I$

$A^5=8A^3+4A^2-8A$

$=8(2A^2+4A-4I)+4A^2-8A$

$A^5=20A^2+24A-32I$

$\therefore \alpha=20, \beta=24, \gamma=-32$

$\therefore \alpha+\beta+\gamma=12$

SECTION-B

21. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}+2y\sec^2 x=2\sec^2 x+3\tan x.\sec^2 x$ such that $y(0)=\frac{5}{4}$. Then $12\left(y\left(\frac{\pi}{4}\right)-e^{-2}\right)$ is equal to __________.

Ans. (21)

Sol. $\text{I.F.}=e^{\int 2\sec^2 x dx}$

$=e^{2\tan x}$

Solution of diff. eq.

$y.e^{2\tan x}=\int e^{2\tan x}(2\sec^2 x+3\tan x.\sec^2 x)dx$

$y.e^{2\tan x}=\int e^{2\tan x}.(2\sec^2 x)dx+\int e^{2\tan x}.(3\tan x.\sec^2 x)dx$

$y.e^{2\tan x}=e^{2\tan x}.2\tan x-\int e^{2\tan x}.2\sec^2 x\times 2\tan x dx+\int e^{2\tan x}.3\tan x.\sec^2 x dx$

$y.e^{2\tan x}=2\tan x.e^{2\tan x}-\int e^{2\tan x}.\tan x\sec^2 x dx$

$y.e^{2\tan x}=2\tan x.e^{2\tan x}-\frac{\tan x.e^{2\tan x}}{2}+\frac{e^{2\tan x}}{4}+C$

$y=2\tan x-\frac{\tan x}{2}+\frac{1}{4}+Ce^{-2\tan x}$

$x=0, y=\frac{5}{4}$

$c=1$

$y\left(\frac{\pi}{4}\right)=\frac{7}{4}+e^{-2}$

Then $12\left(y\left(\frac{\pi}{4}\right)-e^{-2}\right)=12\left(\frac{7}{4}\right)=21$

22. If the sum of the first 10 terms of the series $\frac{4.1}{1+4.1^4}+\frac{4.2}{1+4.2^4}+\frac{4.3}{1+4.3^4}+\ldots$ is $\frac{m}{n}$, where gcd(m, n) = 1, then m + n is equal to ________.

Ans. (441)

Sol. $T_r=\frac{4.r}{1+4.r^4}$

$T_r=\frac{4.r}{(2r^2+2r+1)(2r^2-2r+1)}$

$T_r=\frac{(2r^2+2r+1)-(2r^2-2r+1)}{(2r^2+2r+1)(2r^2-2r+1)}$

$$T_r = \frac{1}{2r^2 - 2r + 1} - \frac{1}{2r^2 + 2r + 1}$$

$$T_1 = \frac{1}{1} - \frac{1}{5}$$

$$T_2 = \frac{1}{5} - \frac{1}{13}$$

$\vdots$

$$T_{10} = \frac{1}{181} - \frac{1}{221}$$

$$S_{10} = 1 - \frac{1}{221} = \frac{220}{221} = \frac{m}{n}$$

$m + n = 441$

23. If $y = \cos\left(\frac{\pi}{3} + \cos^{-1}\frac{x}{2}\right)$, then $(x - y)^2 + 3y^2$ is equal to __________.

Ans. **(3)**

Sol. $y = \cos\left(\cos^{-1}\frac{1}{2} + \cos^{-1}\frac{x}{2}\right)$

$$y = \frac{1}{2} \times \frac{x}{2} - \sqrt{1 - \frac{1}{4}}\sqrt{1 - \frac{x^2}{4}}$$

$4y = x - \sqrt{3}\sqrt{4 - x^2}$

$3(4 - x^2) = x^2 + 16y^2 - 8xy$

$12 - 3x^2 = x^2 + 16y^2 - 8xy$

$4x^2 + 16y^2 - 8xy = 12$

$x^2 + 4y^2 - 2xy = 3$

$x^2 + y^2 - 2xy - 3y^2 = 3$

$(x - y)^2 + 3y^2 = 3$

24. Let A(4, –2), B(1, 1) and C(9, –3) be the vertices of a triangle ABC. Then the maximum area of the parallelogram AFDE, formed with vertices D, E and F on the sides BC, CA and AB of the triangle ABC respectively, is __________.

Ans. **(3)**

Sol. Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 4 & -2 & 1 \\ 1 & 1 & 1 \\ 9 & -3 & 1 \end{vmatrix}$

= 6 square units

Maximum area of AFDE = $\frac{1}{2} \times 6 = 3$ sq. units

25. If the set of all $a \in R - \{1\}$, for which the roots of the equation $(1 - a)x^2 + 2(a - 3)x + 9 = 0$ are positive is $(-\infty, -\alpha] \cup [\beta, \gamma)$, then $2\alpha + \beta + \gamma$ is equal to __________.

Ans. **(7)**

Sol. Both the roots are positive

$D \geq 0$

$4(a - 3)^2 - 4 \times 9(1 - a) \geq 0$

$a^2 - 6a + 9 - 9 + 9a \geq 0$

$a^2 + 3a \geq 0$

$a(a + 3) \geq 0$

$a \in (-\infty, -3] \cup [0, \infty)$ ……(i)

$$-\frac{b}{2a} > 0$$

$$\frac{2(a - 3)}{2(a - 1)} > 0$$

$a \in (-\infty, 1) \cup (3, \infty)$ ……..(ii)

$f(0) = 9 > 0$

Equation (i) $\cap$ (ii)

$a \in (-\infty, -3] \cup [0, 1)$

$2\alpha + \beta + \gamma - 6 + 0 + 1 = 7$