ALLEN® CAREER INSTITUTE OVERSEAS

JEE(MAIN) : OVERSEAS COURSE

(Academic Session : 2021 - 2022)

Test Pattern
JEE(Main)
Class-XII

SAMPLE TEST PAPER

ANSWER KEY

PART-1 PHYSICS

SECTION-I	**Q.**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
	A.	C	A	C	C	C	B	C	D	A	D
	Q.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
	A.	D	C	D	D	A	A	B	C	D	A
SECTION-II	**Q.**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
	A.	1.60	500.00	1.70	6.80	0.50	26.00	60.00	7.14	12.00	396.00

PART-2 CHEMISTRY

SECTION-I	**Q.**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
	A.	B	B	D	D	D	C	C	C	C	D
	Q.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
	A.	C	B	B	B	D	B	C	B	B	A
SECTION-II	**Q.**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
	A.	4.00	3.00	2.00	6.00	8.00	45.00	500.00	100.00	250.00	3.00

PART-3 MATHEMATICS

SECTION-I	**Q.**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
	A.	C	C	B	A	C	B	D	B	D	C
	Q.	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
	A.	D	C	D	C	B	C	B	D	B	C
SECTION-II	**Q.**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
	A.	10.00	18.00	2160.00	17.00	1.00	7.00	1.16	5.00	2.00	108.00

HINT – SHEET

PART-1 : PHYSICS

SECTION-I

1. Reynolds Number = $\frac{\rho v d}{\eta}$

Volume flow rate = v × πr^2

$$v=\frac{100\times10^{-3}}{60}\times\frac{1}{\pi\times25\times10^{-4}}$$

$$v=\frac{2}{3\pi}m/s$$

Reynolds Number = $\frac{10^3\times2\times10\times10^{-2}}{10^{-3}\times3\pi}\simeq2\times10^4$

order 10^4

2. $-ms\frac{dT}{dt}=e\sigma A\left(T^4-T_0^4\right)$

$$-\frac{dT}{dt}=\frac{e\sigma A}{ms}\left(T^4-T_0^4\right)$$

$$-\frac{dT}{dt}=\frac{4e\sigma AT_0^3}{ms}(\Delta T)$$

$T = T_0 + (T_i - T_0)e^{-kt}$

where $k=\frac{4e\sigma AT_0^3}{ms}$

$$k=\frac{4e\sigma AT_0^3}{\rho vs}$$

$$\left|\frac{dT}{dt}\right|\propto k$$

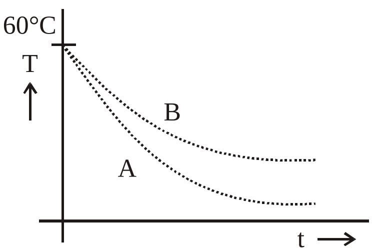
$$\therefore\ \left|\frac{dT}{dt}\right|\propto\frac{1}{\rho s}$$

$\rho_A s_A = 2000 \times 8 \times 10^2 = 16 \times 10^5$

$\rho_B s_B = 4000 \times 10^3 = 4 \times 10^6$

$\rho_A s_A < \rho_B s_B$

$$\left|\frac{dT}{dt}\right|_A>\left|\frac{dT}{dt}\right|_B$$



3. Energy of catapult $=\frac{1}{2}\times\left(\frac{\Delta\ell}{\ell}\right)^2\times Y\times A\times\ell$

= Kinetic energy of the ball $=\frac{1}{2}mv^2$

therefore,

$$\frac{1}{2}\times\left(\frac{20}{42}\right)^2\times Y\times\pi\times3^2\times10^{-6}\times42\times10^{-2}$$

$$=\frac{1}{2}\times2\times10^{-2}\times(20)^2$$

$Y \simeq 3 \times 10^6\ Nm^{-2}$

4. Suppose 'm' gram of water evaporates then, heat required

$\Delta Q_{req} = mL_v$

Mass that converts into ice = (150 – m)

So, heat released in this process

$\Delta Q_{rel} = (150 - m)\ L_f$

Now,

$\Delta Q_{rel} = \Delta Q_{req}$

$(150 - m)\ L_f = mL_V$

$m(L_f + L_v) = 150\ L_f$

$$m=\frac{150L_f}{L_f+L_v}$$

m = 20g

5. $v_{rms}=\sqrt{\frac{3RT}{m}}$ $\qquad v_{escape}=\sqrt{2gR_e}$

$v_{rms} = v_{escape}$

$$\frac{3RT}{m}=2gR_e$$

$$\frac{3\times1.38\times10^{-23}\times6.02\times10^{26}}{2}\times T$$

$= 2 \times 10 \times 6.4 \times 10^6$

$T=\frac{4\times10\times6.4\times10^6}{3\times1.38\times6.02\times10^3}=10\times10^3 = 10^4 k$

Note : Question gives avogadro Number $N_A=6.02\times10^{26}$/kg but we take $N_A=6.02\times10^{26}$/kmol.

6. $A = A_0e^{-\gamma t}$

$A = \frac{A_0}{2}$ after 10 oscillations

$\because$ After 2 seconds

$\frac{A_0}{2} = A_0e^{-\gamma(2)}$

$2 = e^{2\gamma}$

$\ell n2 = 2\gamma$

$\gamma = \frac{\ell n2}{2}$

$\because$ $A = A_0e^{-\gamma t}$

$\ell n\frac{A_0}{A} = \gamma t$

$\ell n1000 = \frac{\ell n2}{2}t$

$2\left(\frac{3\ell n10}{\ell n2}\right) = t$

$\frac{6\ell n10}{\ell n2} = t$

$t = 19.931$ sec

$t \approx 20$ sec

7. Angular impulse = change in angular momentum

$\tau\,\Delta t = \Delta L$

$mg\frac{\ell}{2}\times .01 = \frac{m\ell^2}{3}\omega$

$\omega = \frac{3g\times 0.01}{2\ell}$

$= \frac{3\times 10\times .01}{2\times 0.3}$

$= \frac{1}{2} = 0.5$ rad/s

time taken by rod to hit the ground

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 5}{10}} = 1$ sec.

in this time angle rotate by rod

$\theta = \omega t = 0.5 \times 1 = 0.5$ radian

8. Particles can have same magnitude of momentum but different directions.

9. $N_t = N_0e^{-\lambda t}$

$\lambda nN_t = -\lambda t + \ell n\,N_0$

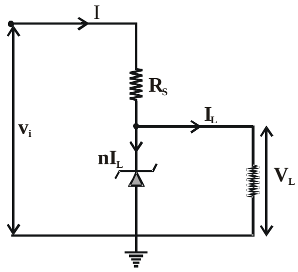
at $t = 0$, $\ell n\,N_t = 3 = \ell n\,N_0$, $N_0 = e^3 = (2.7)^3 = 20$

at $t = 60s$, $\ell n\,N_t = -\lambda \times 60 + 3 = 0$

$\lambda = \frac{3}{20} = \frac{1}{20} = 0.05/s$

$N_t = N_0e^{-\lambda t} = 20\,e^{-0.05t/s}$

10. $V_i - V_L = IR_S$



$V_i - V_L = (nI_L + I_L)R_S$

$R_S = \frac{V_i - V_L}{(n+1)I_L}$

11. At time t

$\frac{N_B}{N_A} = .3 \Rightarrow N_B = .3\,N_A$

also let initially there are total N_0 number of nuclei

$N_A + N_B = N_0$

$N_A = \frac{N_0}{1.3}$

Also as we know

$N_A = N_0e^{-\lambda t}$; $\frac{N_0}{1.3} = N_0e^{-\lambda t}$

$\frac{1}{1.3} = e^{-\lambda t} \Rightarrow \ell n(1.3) = \lambda t$ or $t = \frac{\ell n(1.3)}{\lambda}$

$t = \frac{\ell n(1.3)}{\frac{\ell n(2)}{T}} = \frac{\ell n(1.3)}{\ell n(2)}T$

SECTION-II

1. $M = \int_0^R \rho_0 r(2\pi r dr) = \frac{\rho_0\times 2\pi\times R^3}{3}$

$\underset{\text{(MOI about COM)}}{I_0} = \int_0^R \rho_0 r(2\pi r dr)\times r^2 = \frac{\rho_0\times 2\pi R^5}{5}$

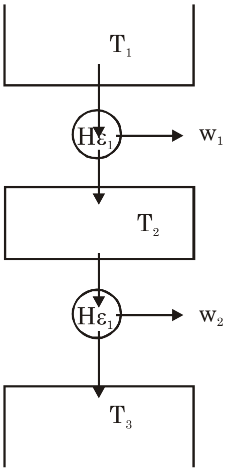
by parallel axis theorem

$I = I_0 + MR^2$

$= \frac{\rho_0\times 2\pi R^5}{5} + \frac{\rho_0\times 2\pi R^3}{3}\times R^2 = \rho_0 2\pi R^5\times\frac{8}{15}$

$= MR^2\times\frac{8}{5}$

2.



$w_1 = w_2$

$\Delta u_1 = \Delta u_2$

$T_3 - T_2 = T_2 - T_1$

$2T_2 = T_1 + T_3$

$T_2 = 500$ K

3. Given

$\frac{Y_A}{Y_B} = \frac{7}{4}$ $\quad L_A = 2m \quad A_A = \pi R^2$

$L_B = 1.5m \quad A_B = \pi(2mm)^2$

$\frac{F}{A} = Y\left(\frac{\ell}{L}\right)$

given F and ℓ are same $\Rightarrow \frac{AY}{L}$ is same

$\frac{A_A Y_A}{L_A} = \frac{A_B Y_B}{L_B}$

$\Rightarrow \frac{(\pi R^2)\left(\frac{7}{4}Y_B\right)}{2} = \frac{\pi(2mm)^2 . Y_B}{1.5}$

R = 1.74 mm

4. $T = \frac{30\,sec}{20}$ $\quad \Delta T = \frac{1}{20}$ sec.

L = 55 cm $\quad \Delta L = 1mm = 0.1$ cm

$g = \frac{4\pi^2 L}{T^2}$

percentage error in g is

$\frac{\Delta g}{g} \times 100\% = \left(\frac{\Delta L}{L} + \frac{2\Delta T}{T}\right)100\%$

$= \left(\frac{0.1}{55} + \frac{2\left(\frac{1}{20}\right)}{\frac{30}{20}}\right)100\% \simeq 6.8\%$

5. $\frac{hc}{\lambda} = W_0 + \frac{1}{2}mv_{max}^2$

Assuming W_0 to be negligible in comparison to $\frac{hc}{\lambda}$

i.e. $v_{max}^2 \propto \frac{1}{\lambda} \Rightarrow v_{max} \propto \frac{1}{\sqrt{\lambda}}$

(On increasing wavelengthλ to 4λ , v_{max} becomes half).

6. $\frac{r_n}{r_{He}} = \left(\frac{A}{4}\right)^{1/3} = (14)^{1/3}$

$\Rightarrow \quad A = 56$

Given $\quad N = 30$

$\therefore \quad Z = A - N = 56 - 30 = 26$

10. $n \propto \frac{1}{\ell} \Rightarrow n_1\ell_1 = n_2\ell_2 \Rightarrow (n+4)49 = (n-4)50$

$\Rightarrow n = 396$

PART-2 : CHEMISTRY

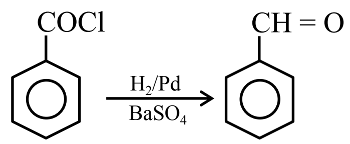
SECTION-I

11. C — O $\quad>\quad$ C — O $\quad>\quad$ C — O

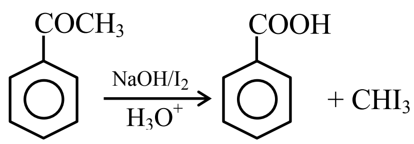
Pure single $\quad$ Impure single $\quad$ Impure double

(z) $\quad$ (y) $\quad$ (x)

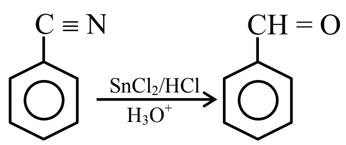
13.



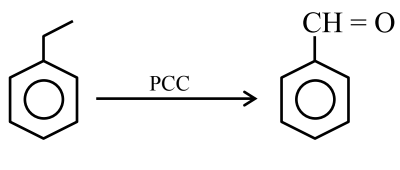
Rosenmund reduction



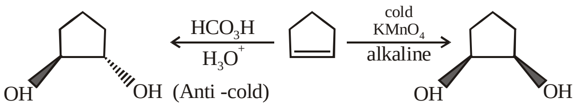
Iodoform reaction



Stephun reaction



Mild oxidation

14. 

16. $N_2O_4 \rightleftharpoons 2NO_2$

1 0

$1 - x$ $2x = 1 + x$

$= 1 - \alpha$ $2\alpha = 1 + \alpha$

$$kp = \frac{\left(\frac{2\alpha}{1+\alpha}\times p\right)^2}{\left(\frac{1-\alpha}{1+\alpha}\times p\right)}$$

$$k_p = \frac{4\alpha^2.P}{1-\alpha^2}$$

$K_p - K_p\,\alpha^2 = 4k^2.p$

$\alpha^2\,(4p + kp) = Kp$

$$\alpha = \sqrt{\frac{k_p}{4_p + k_p}}$$

17. $$\frac{\Delta T_b}{\Delta T_f} = \frac{k_b \times m}{k_f \times m}$$

$$\frac{1}{\Delta T_f} = \frac{0.52\times m}{1.86\times m}$$

$$\Delta T_f = \frac{1.86}{0.52} = 3.61°C$$

$\because$ $(F.P)_{solution} = -3.61$ C

18. $\Delta H = \Delta U + \Delta ngRT$

$$-3271 = \Delta U + \frac{(-1.5)\times 8.314\times 300}{1000}$$

$\Delta U = -3267.25$ KJ/mol

for 1 mole of bezene

ΔU for 1.5 mol of benzene

$= -3267.25 \times 1.5$

$= -4900.25$ KJ

19. $$pH = pK_a + \log\frac{1/3}{2/3}$$

20. $$pH = pK_a + \log\frac{[HCO_3^-]}{[H_2CO_3]}$$

or $$7.4 = 6.1 + \log\frac{[HCO_3^-]}{[H_2CO_3]}$$

or $$\frac{[HCO_3^-]}{[H_2CO_3]} = 20$$

SECTION-II

7. $$t_{x\%} = \frac{2.303}{K}\log\frac{100}{100-x}$$

$$= \frac{2.303\ t_{1/2}}{\ln 2}\log\frac{100}{(100-99.999)}$$

$$= \frac{2.303\times t_{1/2}}{2.303\log 2}\log\frac{100}{10^{-3}}$$

$$\frac{30}{0.3}\log 10^5 = \frac{30\times 5}{0.3} = 500$$

8. $$\frac{\overset{o}{P} - P_s}{\overset{o}{P}} = x_{solute}$$

$$\frac{\overset{o}{P} - 80}{\overset{o}{P}} = 0.2$$

$\overset{o}{P} = 100$ torr.

9. $nC_v(T_2 - T_1) = -75$ cal

$$0.1 \times \frac{3R}{2}(T_2 - T_1) = -75$$

$$T_2 - T_1 = \frac{-75}{0.3}$$

$T_2 - 500 = -250$

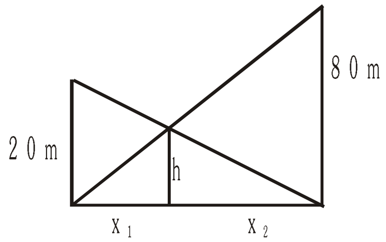
$T_2 = 250$ K.

10. $$[H^+] = C\alpha = 0.1 \times \frac{1}{100} = 10^{-3}\ M$$

$pH = -\log[H^+] = -\log 10^{-3} = 3$

PART-3 : MATHEMATICS

SECTION-I

1. 

by similar triangle

$$\frac{h}{x_1} = \frac{80}{x_1 + x_2} \quad ...(1)$$

by $$\frac{h}{x_2} = \frac{20}{x_1 + x_2} \quad ..(2)$$

by (1) and (2)

$\frac{x_2}{x_1} = 4$ or $x_2 = 4x_1$

$$\Rightarrow \frac{h}{x_1} = \frac{80}{5x_1}$$

or h = 16 m

2. $(p \to q) \wedge (q \to \sim p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee \sim p)$

$\equiv \sim p \vee (q \wedge \sim q)$

$\equiv \sim p \vee C \equiv \sim p$

3. $|M| \neq 0$ [M is invertible]

$\text{adj}(\text{adj}M) = |M|\, M = M^{-1}$ .....(1)

$\Rightarrow |M|MM = M \cdot M^{-1}$

$\Rightarrow |M| \cdot M^2 = I$

$\Rightarrow |M|\, |M^2| = I$

$\Rightarrow |M|^5 = |I| \Rightarrow |M| = 1$

From

(1) $M^{-1} = M$

$\Rightarrow I = M^2$

$\text{adj}(M^2) = \text{adj}(I) = I$

4. $\frac{2p}{3} = \frac{3q}{5} = \frac{4r}{7} = k$

$p = \frac{3k}{2},\ q = \frac{5k}{3},\ r = \frac{7k}{4}$

$p : q : r = \frac{3k}{2} : \frac{5k}{3} : \frac{7k}{4}.$

$= 18 : 20 : 21$

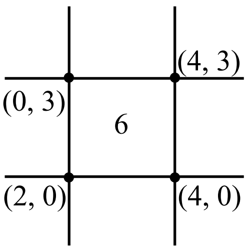
6. $2x^3 + 5x^2 + 2x - 1 = (x + 1)(2x^2 + 3x - 1)$

$\Rightarrow$ either $x = -1$ is root of equation

or $x^2 + ax + b = 0$ & $2x^2 + 3x - 1 = 0$ have both roots in common.

7. $x \geq 2, \quad x \leq 4,$

$y \geq 2, \quad y \leq 3$



9. $|f(x)| = \begin{cases} 1 & , \ -2 \leq x < 0 \\ 1 - x^2 & , \ 0 \leq x < 1 \\ x^2 - 1 & , \ 1 \leq x \leq 2 \end{cases}$

and $f(|x|) = x^2 - 1,\ x \in [-2, 2]$

Hence $g(x) = \begin{cases} x^2 & , \ x \in [-2, 0) \\ 0 & , \ x \in [0, 1) \\ 2(x^2 - 1) & , \ x \in [1, 2] \end{cases}$

It is not differentiable at x = 1

10. Differentiating with respect to x,

$x.\frac{1}{\ell nx}.\frac{1}{x} + \ell n(\ell nx) - 2x + 2y.\frac{dy}{dx} = 0$

at x = e we get

$1 - 2e + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{2e - 1}{2y}$

$\Rightarrow \frac{dy}{dx} = \frac{2e - 1}{2\sqrt{4 + e^2}}$ as $y(e) = \sqrt{4 + e^2}$

11. $y = x^2 - 5x + 5$

$\frac{dy}{dx} = 2x - 5 = 2 \Rightarrow x = \frac{7}{2}$

at $x = \frac{7}{2},\ y = \frac{-1}{4}$

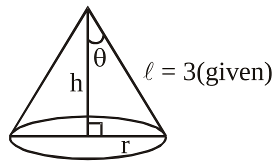
Equation of tangent at $\left(\frac{7}{2}, \frac{-1}{4}\right)$ is

$2x - y - \frac{29}{4} = 0$

Now check options

$x = \frac{1}{8},\ y = -7$

12.



$\therefore h = 3 \cos \theta$

$r = 3 \sin \theta$

Now,

$V = \frac{1}{3}\pi r^2 h = \frac{\pi}{3}(9\sin^2\theta).(3\cos\theta)$

$\therefore \frac{dV}{d\theta} = 0 \Rightarrow \sin\theta = \sqrt{\frac{2}{3}}$

Also, $\left.\frac{d^2V}{d\theta^2}\right]_{\sin\theta = \sqrt{\frac{2}{3}}}$ = negative

$\Rightarrow$ Volume is maximum,

when $\sin\theta = \sqrt{\frac{2}{3}}$

$\therefore V_{max}\left(\sin\theta = \sqrt{\frac{2}{3}}\right) = 2\sqrt{3}\pi$ (in cu. m)

13. Let $x^{1/6} = t$

$\int\left(\frac{t^6 + t^4 + t}{t^6(1 + t^2)}\right)6t^5 dt = 6\left(\tan^{-1} t + \frac{t^4}{4}\right) + c$

14. $I = \int_{-\frac{1}{8}}^{\frac{1}{8}} \cos^{-1}\left(x^5 + 5x\right) dx$

$2I = \int_{-\frac{1}{8}}^{\frac{1}{8}} \pi \, dx \qquad \therefore I = \frac{\pi}{8}$

15. $\int \frac{y\,dy}{1+y^2} = \int \frac{1+x+x^2}{x\left(1+x^2\right)} dx$

$\frac{1}{2} \ell n\left(1+y^2\right) = \tan^{-1} x + \ell n\, x + c$

$\Rightarrow \ell n\left(\frac{1+y^2}{x^2}\right) = 2\tan^{-1}x + c$

when $x = 1$, $y = 0$ $\therefore c = -\pi/2$

SECTION-II

1. $\tan\alpha + \tan\beta = \frac{\lambda\sqrt{2}}{k+1}$

$\tan\alpha \, . \, \tan\beta = \frac{k-1}{k+1}$

$\tan(\alpha+\beta) = \frac{\frac{\lambda\sqrt{2}}{k+1}}{1-\frac{k-1}{k+1}} = \frac{\lambda\sqrt{2}}{2} = \frac{\lambda}{\sqrt{2}}$

$\Rightarrow \frac{\lambda^2}{2} = 50 \Rightarrow \lambda = 10 \,\&\, -10$

2. Variance of first 'n' natural numbers

$= \frac{n^2-1}{12} = 10 \Rightarrow n = 11$

and variance of first 'm' even natural numbers

$= 4\left(\frac{m^2-1}{12}\right) \Rightarrow \frac{m^2-1}{3} = 16 \Rightarrow m = 7$

$m + n = 18$

4. $x + y + z = 10$

Total no. of non negative sol.= 10 + 3 – 1

For C_{3-1}

No. of cases in which z is even = ${}^{12}C_2 = 66$

Let $z = 2K$

$x + y = 10 - 2K$

$x, y \geq 0$

No. of solution = 10 – 2K + 2 – 1

C_{2-1}

= 11 – 2K

Now

11 – 2K = 11 + 9 + 7 + 5 + 3 + 1 = 36

$\text{Prof} = \frac{36}{66} = \frac{6}{11} = \frac{p}{q}$

p + q = 17

5. $\lim_{x\to 0} \frac{x\tan^2 2x}{\tan 4x \sin^2 x} = \lim_{x\to 0} \frac{x\left(\frac{\tan^2 2x}{4x^2}\right)4x^2}{\left(\frac{\tan 4x}{4x}\right)4x\left(\frac{\sin^2 x}{x^2}\right)x^2} = 1$

6. $f'(x) = 3x^2 - 6(a-2)x + 3a$

$f'(x) \geq 0 \; \forall \; x \in (0, 1]$

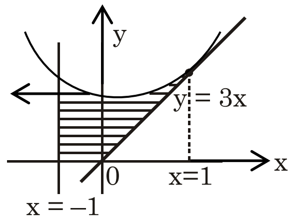
$f'(x) \leq 0 \; \forall \; x \in [1, 5)$

$\Rightarrow f'(x) = 0$ at $x = 1 \Rightarrow a = 5$

$f(x) - 14 = (x-1)^2 (x-7)$

$\frac{f(x)-14}{(x-1)^2} = x - 7$

7.



$= \int_{-1}^{0} \left(x^2+x+1\right) dx + \int_{0}^{1} \left[\left(x^2+x+1\right) - 3x\right] dx$

$= \frac{7}{6}$

8.

$\int_0^1 0\,dx + \int_1^2 (x-1)\,dx + \int_2^3 2(x-2)\,dx + \int_3^4 3(x-3)\,dx + \int_4^5 4(x-4)\,dx$

= 5.00

9. Here $C_1C_2 = \sqrt{47}$, $r_1 + r_2 = 8$ and $|r_1 - r_2| = 2$

$\Rightarrow |r_1 - r_2| < C_1C_2 < r_1 + r_2 \Rightarrow$ two common tangents.

10. Normal at $P(t_1^2, 2t_1)$ on the parabola $y^2 = 4x$ ...(i)

Meets it again at the point $Q\left(t_2^2, 2t_2\right)$,

where $t_2 = -t_1 - \frac{2}{t_1}$ ...(ii)

If subtends a right angle at the vertex (0, 0) then (Slope of OP) (Slope of OQ) = –1

$\Rightarrow \frac{2t_1}{t_1^2} \cdot \frac{2t_2}{t_2^2} = -1 \Rightarrow t_2 = \frac{-4}{t_1}$ ...(iii)

From (ii) and (iii), $-t_1 - \frac{2}{t_1} = \frac{-4}{t_1}$

$\Rightarrow -t_1 = -\frac{2}{t_1}$

$\Rightarrow t_1^2 = 2 \Rightarrow t_1 = \pm\sqrt{2} \;\therefore\; t_2 = \mp 2\sqrt{2}$

$\therefore$ P and Q are $(2, \pm 2)$ and $(8, \mp 4\sqrt{2})$

$\therefore PQ = \sqrt{(8-2)^2 + (\mp 4\sqrt{2} \mp 2\sqrt{2})^2} = \sqrt{36+72}$

$= \sqrt{108} = 6\sqrt{3}$.